Optical clocks and geodesy

Pacôme DELVA
(Pacome.Delva@obspm.fr)

SYRTE, Observatoire de Paris, PSL Research University, CNRS, Sorbonne Universités, UPMC Univ. Paris 06, LNE

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Outline

1. What is chronometric geodesy?
2. Proof-of-principle of chronometric geodesy
3. Some definitions and conventions
4. ACES/Pharao
5. Unifying the gravitational redshift correction
6. Chronometric geodesy for high resolution geopotential

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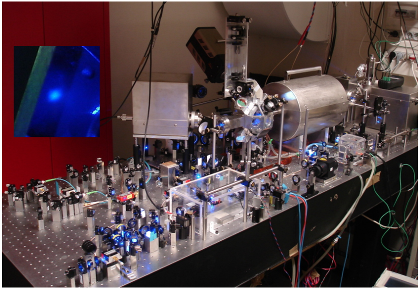
- Chronometric geodesy is sometimes named clock-based geodesy



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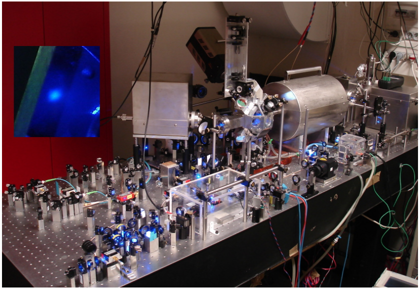

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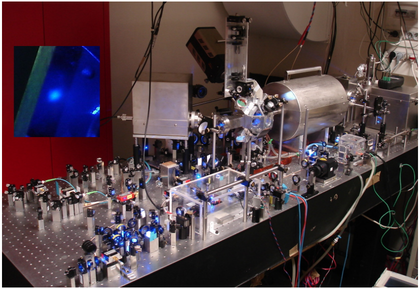

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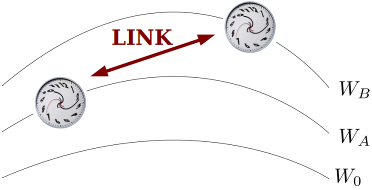
Basic principle of chronometric geodesy

The flow of time, or the rate of a clock when compared to coordinate time, depends on the velocity of the clock and on the space-time metric (which depends on the mass/energy distribution).

In the weak-field approximation:

$$\frac{\Delta\tau}{\tau} = \frac{\Delta f}{f} = \frac{U_B - U_A}{c^2} + \frac{v_B^2 - v_A^2}{2c^2} + O(c^{-4})$$
$$= \frac{W_B - W_A}{c^2} + O(c^{-4}) \quad (1)$$



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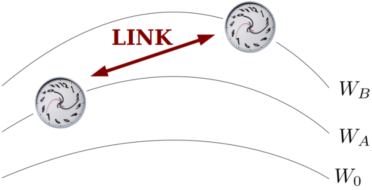

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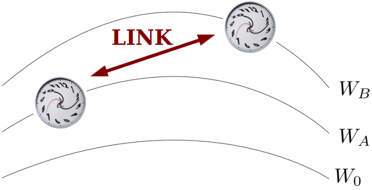

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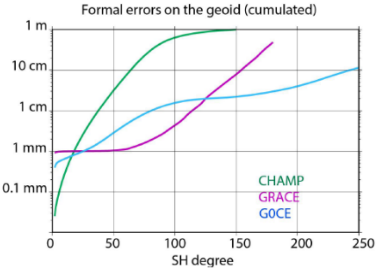
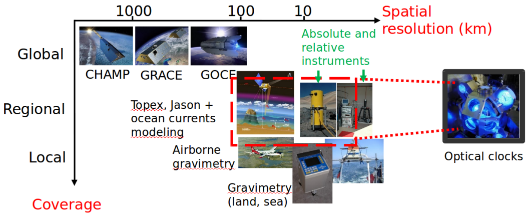
Chronometric observables in geodesy

- Chronometric observables are a completely new type of observable in geodesy: gravity potential differences are directly observed
- Accuracy of optical clocks starts to be competitive with classical methods which have accuracies up to a few centimeters for the static potential at high spatial resolution





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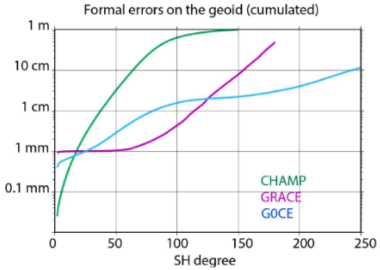

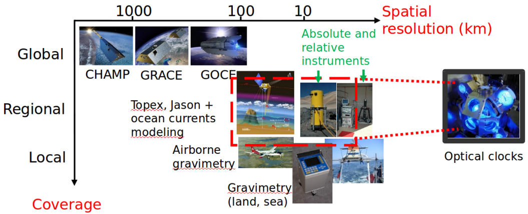

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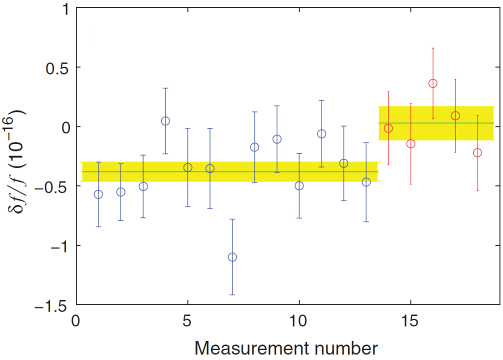
A local comparison

Experimental demonstration of the dependency of clock frequency with local heigth [Chou et al., 2010] with two Al^+ optical clocks.

Starting at data point 14, one of the clock is elevated by 33 cm. The net relative shift is measured to be $(41 \pm 16) \times 10^{-18}$.



The shape of the Earth

As a proof-of-principle, one can determine (roughly) J_2 with two clocks:

$$\frac{\Delta f}{f} = \frac{W_B - W_A}{c^2} + O(c^{-4}) \; , \; W = U + \frac{v^2}{2}$$

$$U = \frac{GM_E}{r}\left[1 + \frac{J_2 R_E^2}{2r^2}\left(1 - 3\sin(\phi)^2\right)\right]$$

- using INRIM CsF1 vs. SYRTE FO2 comparison we find:

$$J_2 = (1.097 \pm 0.016) \times 10^{-3}$$

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Possibilities for technical realisation of a system for measuring potential differences over intercontinental distances using clock comparisons [Vermeer, 1983]

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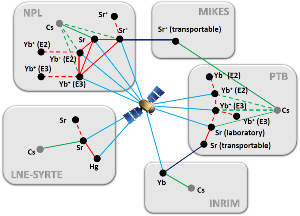
EMRP
European Metrology Research Programme
Programme of EURAMET

The EMRP is jointly funded by the EMRP participating countries within EURAMET and the European Union

ITOC
International Timescales with Optical Clocks



Main challenge: stable links for frequency comparison

- Satellite (GNSS, TWSTFT): intercontinental but limited to $\sim 10^{-16}$, rather long integration time
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Isochronometric surfaces

- An isochronometric surface S is a surface where all clocks beat at the same rate:

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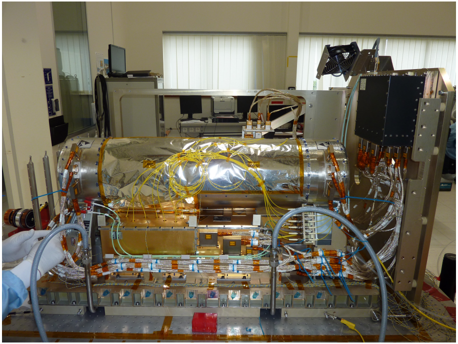
ACES-Paharao: une fontaine atomique dans l'espace



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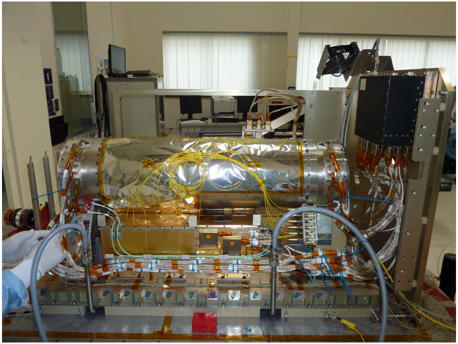

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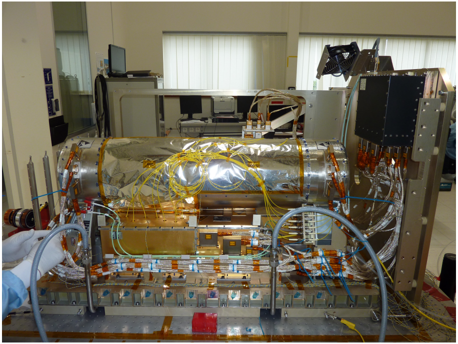

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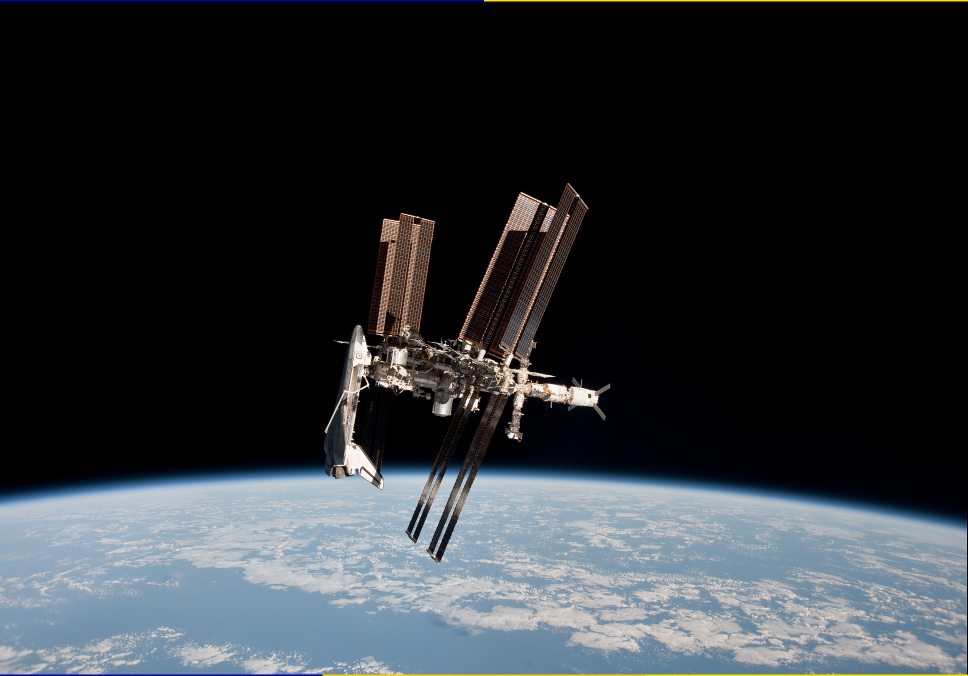
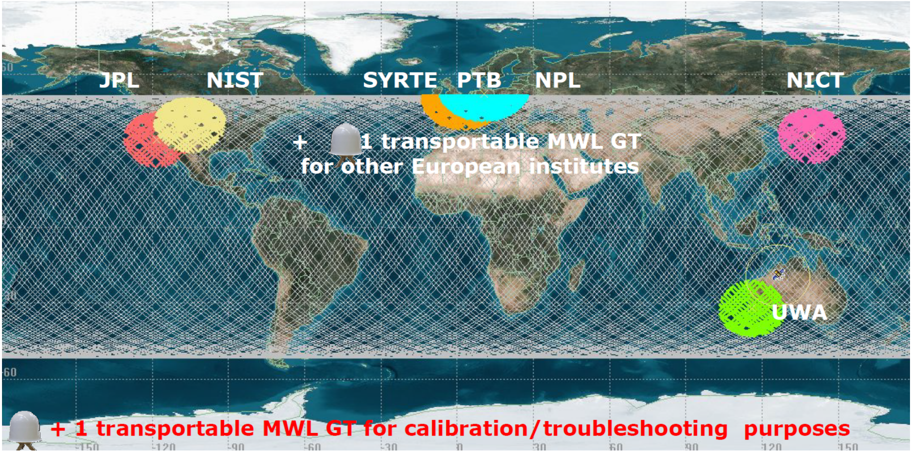


Les stations au sol



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Consortium of the ITOC EMRP project

National Physical Laboratory (NPL, UK)

Cesky Metrologicky Institut (CMI, Czech Republic)

Istituto Nazionale di Ricerca Metrologica (INRIM, Italy)

Mittatekniikan Keskus (MIKES, Finland)

Physikalisch-Technische Bundesanstalt (PTB, Germany)

SYRTE – Paris Observatory (France)

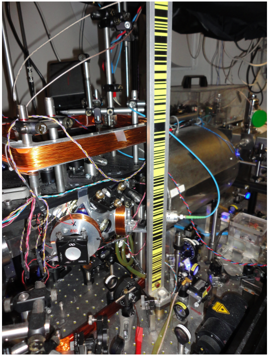
IfE – Leibniz Universität Hannover (Germany)

Classical levelling of the clocks [Denker, 2013]

- Design of setups to determine the static gravity potential at all clock locations
- Development of a refined European geoid model including new gravity observations around all relevant clock sites (done by IfE/LUH)

SYRTE clocks leveling campaign

(IGN SGN Travaux Spéciaux)



Gravity campaigns [Denker, 2015]

NMI	Absolute gravity pts	Relative gravity pts	Comparison with existing pts	Mean difference (mGal)
INRIM	1	35	11	-0.87
LSM	1	122	Italy 10	+0.20
			France 6	+0.37
NPL	2	64	25	-0.07
SYRTE	1+2	97	27	-0.12
PTB	1	45	16	+0.08

A new geoid of reference: EGG2015 [Denker, 2015]

NMI	Mean	Std	Min	Max
INRIM	2.7	1.3	0.7	7.3
LSM	3.9	3.9	-5.3	16.6
NPL	-0.9	0.2	-1.4	0.4
OBSPM	-0.7	0.2	-1.4	-0.2
PTB	-0.3	0.1	-0.6	0.0

Table : Difference statistics between the new (EGG2015) and old (EGG2008) quasigeoid heights in centimers.

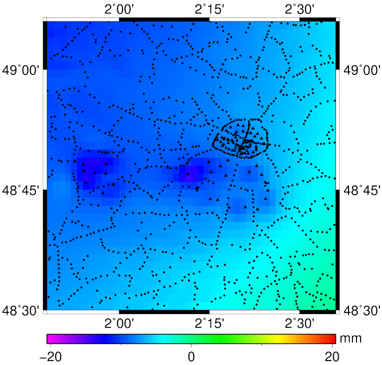


Figure : Differences around OBSPM (▲: new ITOC points; •: old points)

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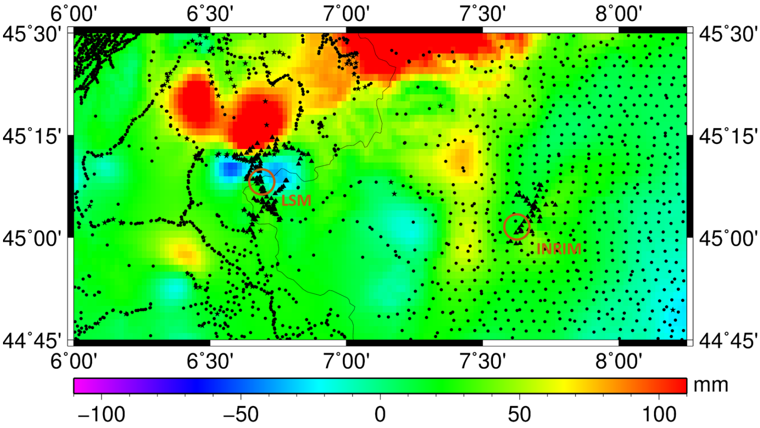
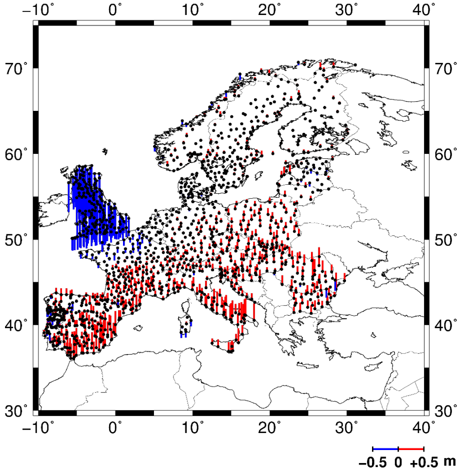


Figure : Differences between the new (EGG2015) and old (EGG2008) quasigeoid heights around INRIM and LSM (▲: new ITOC points; •: old points)

Differences between GNSS/geoid & geometric levelling approach [Denker, 2015]

$$\Delta C = C^{(\mathrm{GNSS/geoid})} - C^{(\mathrm{lev})}$$

NMI	ΔC (10^{-2} $m^2.s^{-2}$)
INRIM	2.3
LSM	-8.3
NPL	-15.3
OBSPM	-11.3
PTB	-2.3



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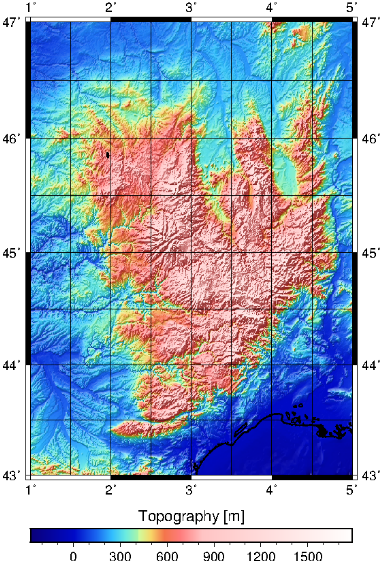
The Auvergne region in France



Interesting region because:

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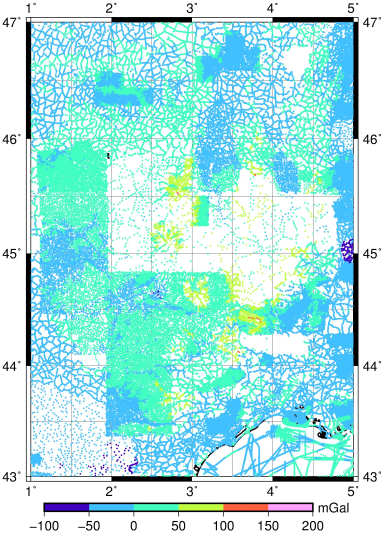

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Global methodology

STEP 1: build synthetic field model → **Reference model**
gravity anomaly $\overline{\delta g}$
potential anomaly $\overline{\delta T}$

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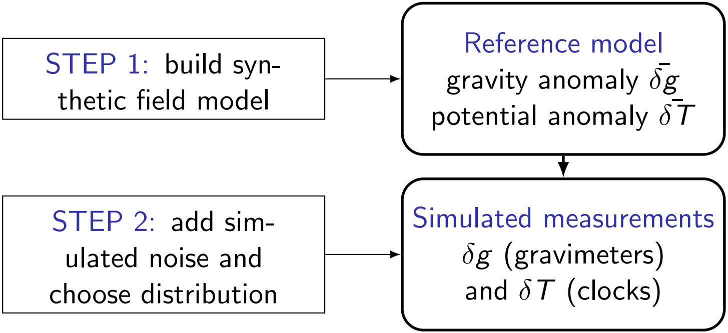

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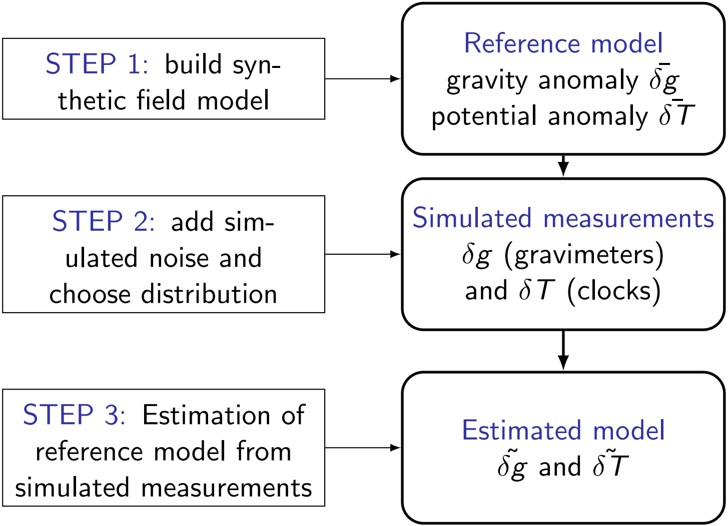

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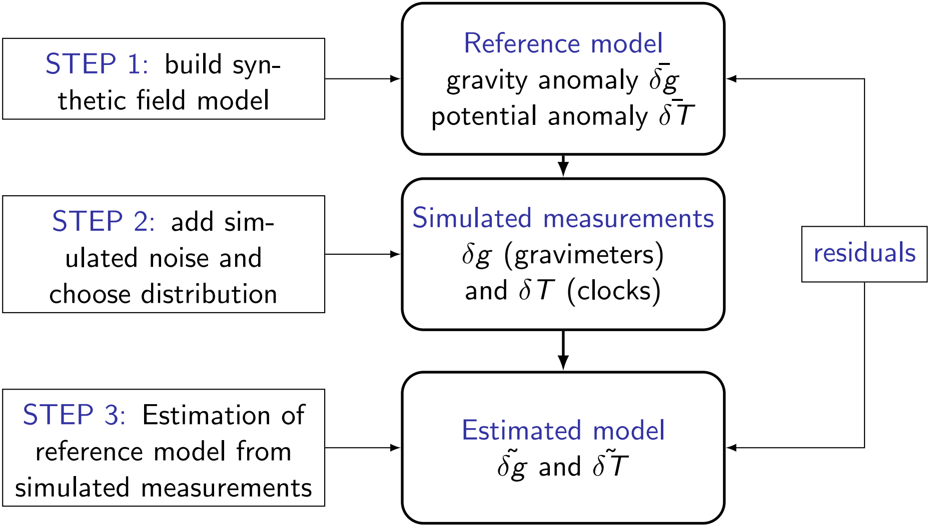

STEP 1: build synthetic field model

1. Global gravity model at 10 km resolution (EIGEN-6C4, Förste et al. 2014)
2. Removal of low frequencies (covered by satellites)

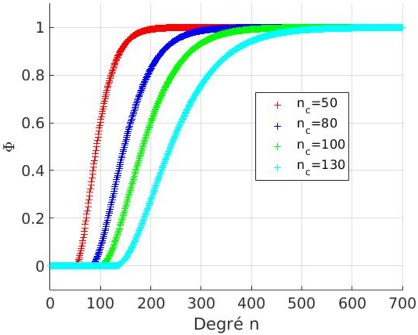

Figure : Filtre based on Poisson wavelets at order 3 (Holschneider et al. 2003)

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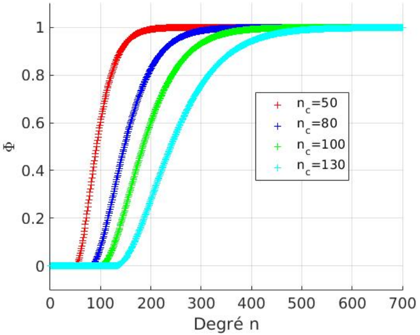

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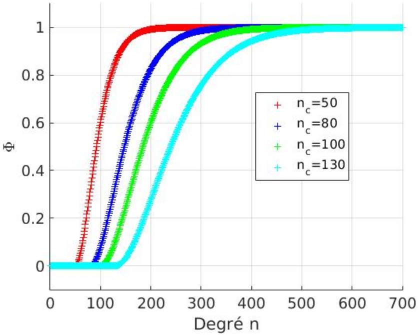

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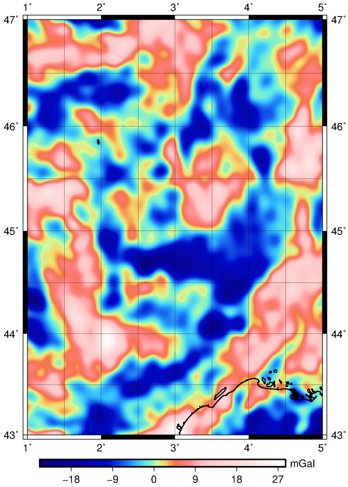
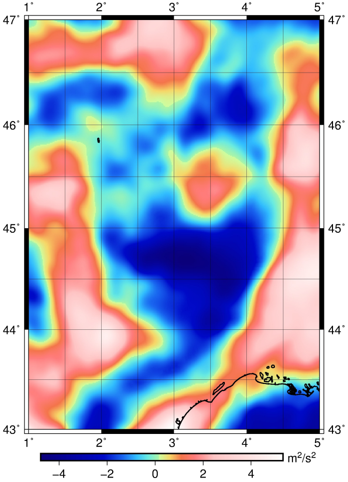


Figure : Reference gravity anomaly



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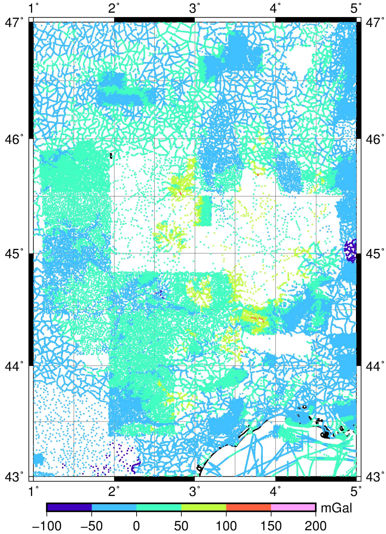
STEP 2: add noise and choose observables distribution



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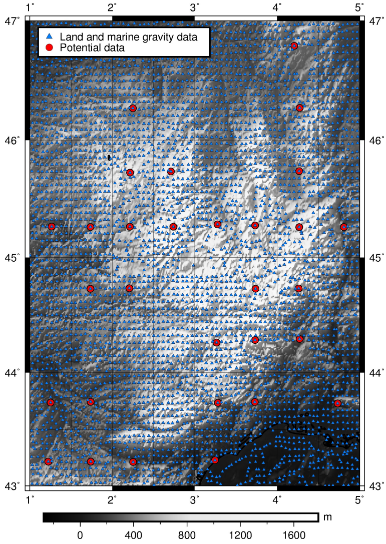

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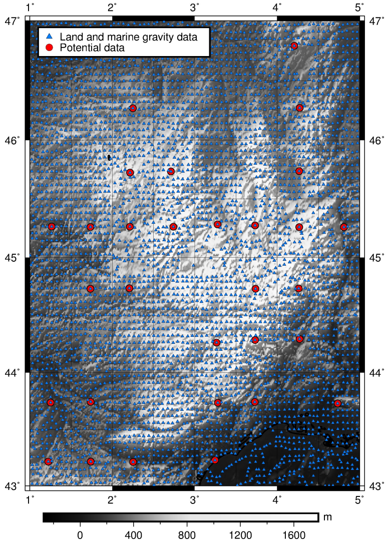

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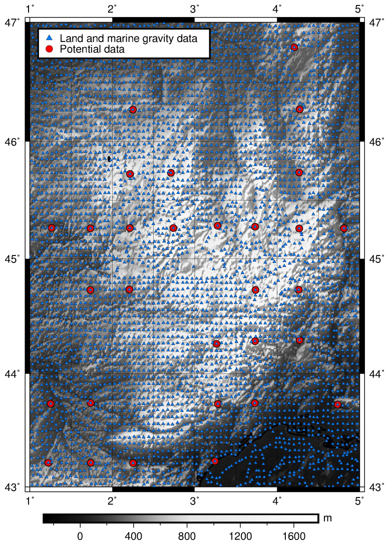

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1. Prior on field regularity: estimation of a 3D covariance function from the simulated gravimetric measurements
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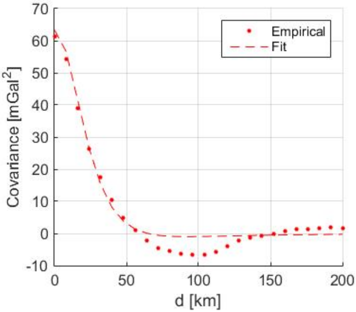

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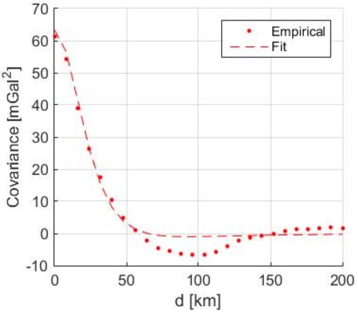

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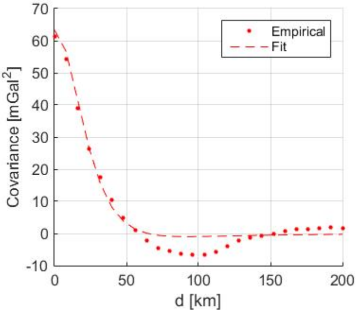

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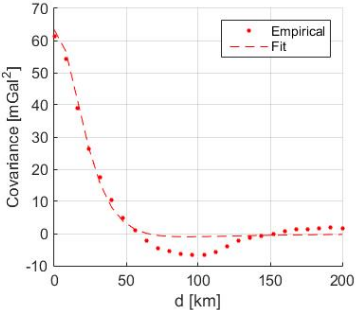

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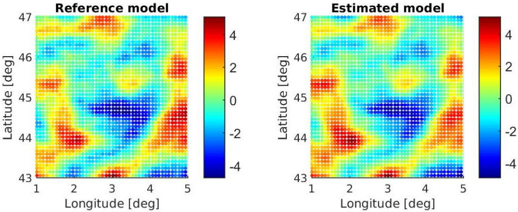

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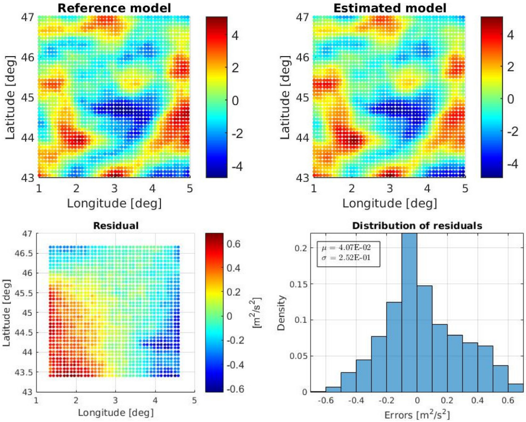

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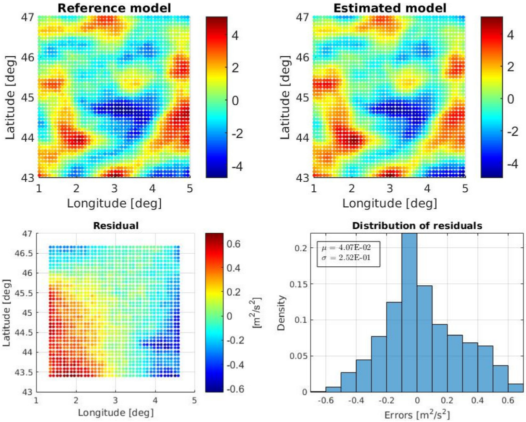

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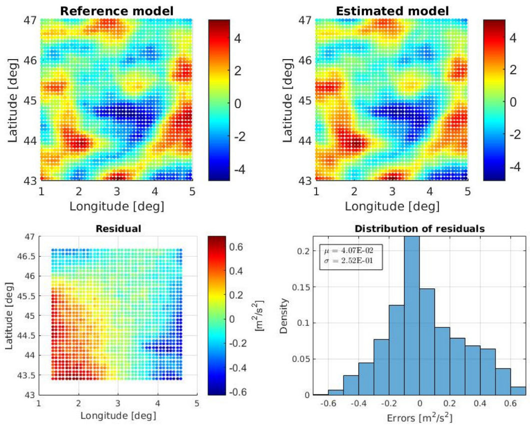

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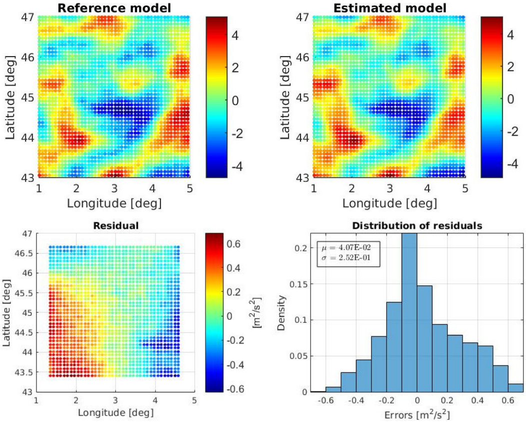

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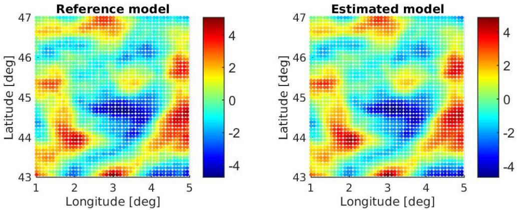

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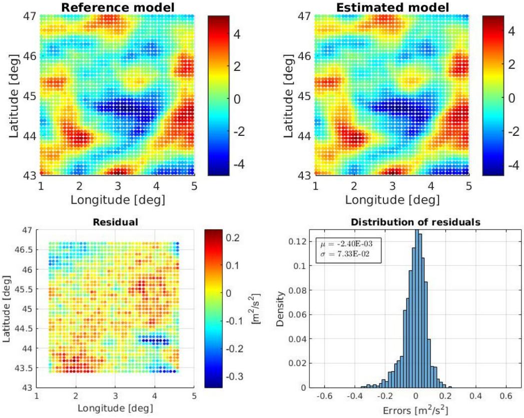

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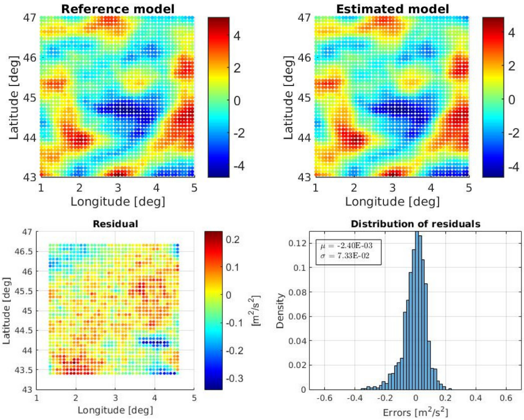

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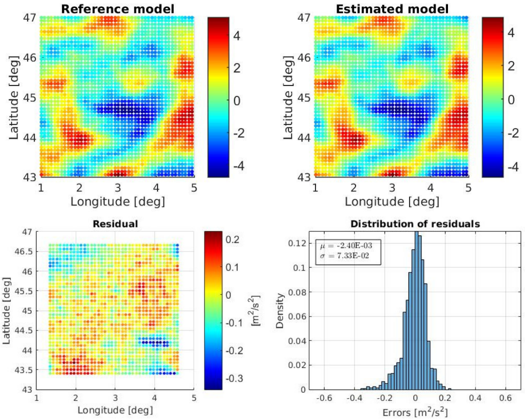

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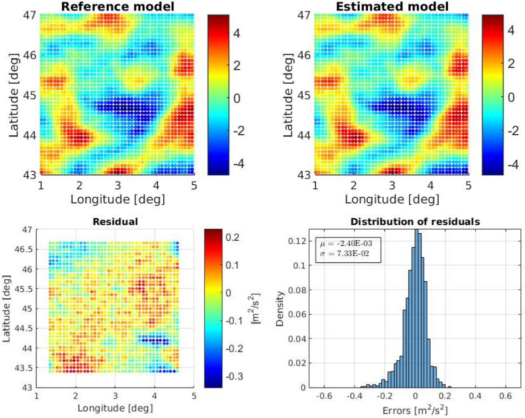

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