

La gravimétrie dans le SI révisé

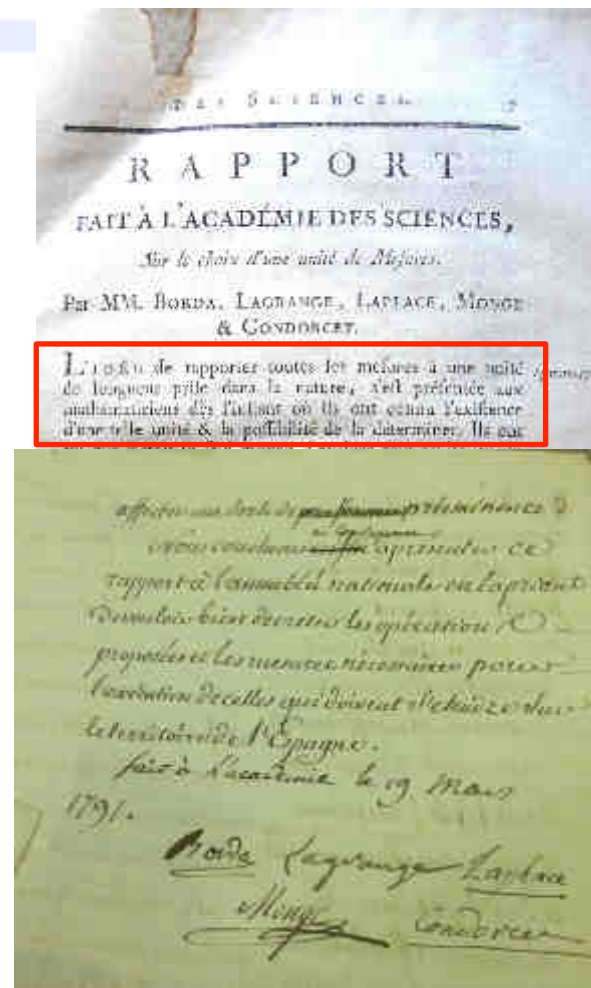
S. Merlet

LNE-SYRTE

*Observatoire de Paris, Université PSL, CNRS, Sorbonne Université,
61 avenue de l'Observatoire, 75014 Paris, France*

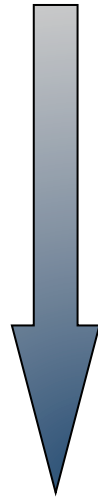
<https://syрте.obspm.fr/spip/science/iaci/>

Systeme International d'Unités (SI)



19 mars 1791

kilogramme, mètre, (seconde)



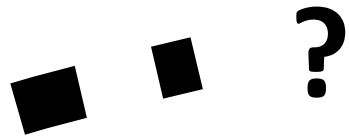
16 novembre 2018

h , c , $\Delta\nu_{Cs}$



.... 20 mai 2019

Histoire du kilogramme



h
2018

1901



1889

Convention
du mètre

1875

1799



XV

1791



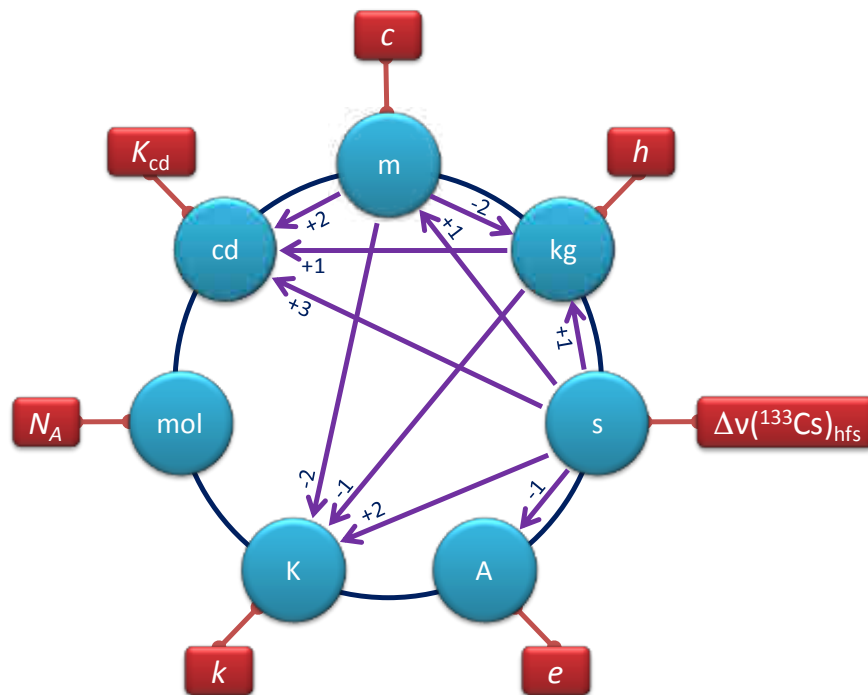
CGPM 2018

decides that, effective from 20 May 2019, the International System of Units, the SI, is the system of units in which:

- ♦ the unperturbed ground state hyperfine transition frequency of the caesium 133 atom $\Delta\nu_{\text{Cs}}$ is 9 192 631 770 Hz,
- ♦ the speed of light in vacuum c is 299 792 458 m/s,

→ Atomic clock

$c = 299\,792\,458$ metres per second



↑
Numerical value

↑
unit, with s already fixed

<https://www.bipm.org/utils/common/pdf/CGPM-2018/26th-CGPM-Resolutions.pdf>

CGPM 2018

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- ♦ the unperturbed ground state hyperfine transition frequency of the caesium 133 atom $\Delta\nu_{\text{Cs}}$ is 9 192 631 770 Hz,
- ♦ the speed of light in vacuum c is 299 792 458 m/s,
- ♦ the Planck constant h is $6.626\,070\,15 \times 10^{-34}$ J s,



Atomic clock

$$c = 299\,792\,458 \text{ metres per second}$$
$$h = 6.626\,070\,15 \times 10^{-34} \text{ kg}\cdot\text{m}^2\cdot\text{s}^{-1}$$



Numerical value



$\text{kg}\cdot\text{m}^2\cdot\text{s}^{-1}$

unit, with
 $\text{m}^2\cdot\text{s}^{-1}$ already fixed

We fixed the size of mass.

where the hertz, joule, coulomb, lumen, and watt, with unit symbols Hz, J, C, lm, and W, respectively, are related to the units second, metre, kilogram, ampere, kelvin, mole, and candela, with unit symbols s, m, kg, A, K, mol, and cd, respectively, according to $\text{Hz} = \text{s}^{-1}$, $\text{J} = \text{kg m}^2 \text{s}^{-2}$, $\text{C} = \text{A s}$, $\text{lm} = \text{cd m}^2 \text{m}^{-2} = \text{cd sr}$, and $\text{W} = \text{kg m}^2 \text{s}^{-3}$.

<https://www.bipm.org/utils/common/pdf/CGPM-2018/26th-CGPM-Resolutions.pdf>

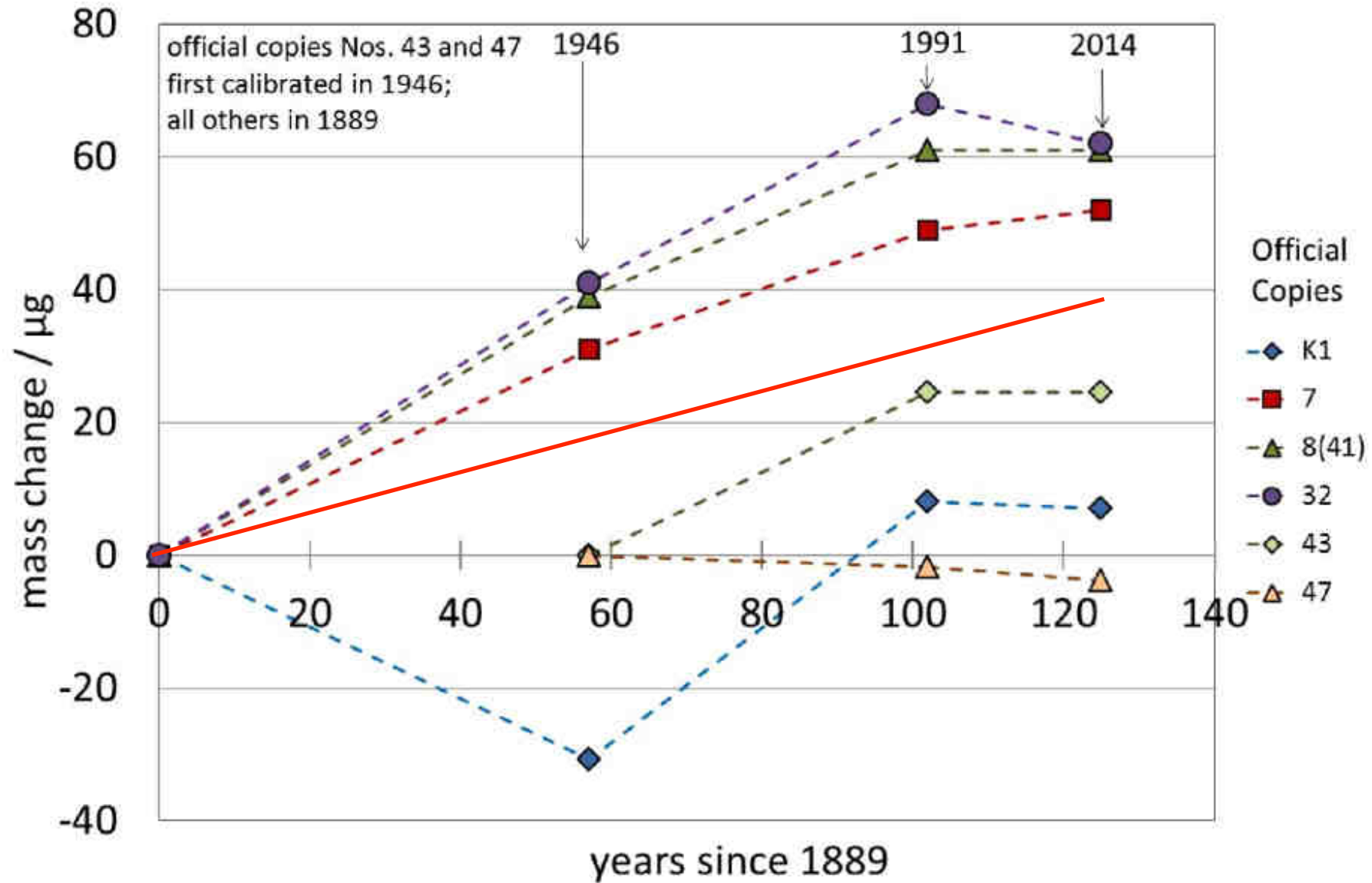
Sommaire

Notre participation dans ce changement

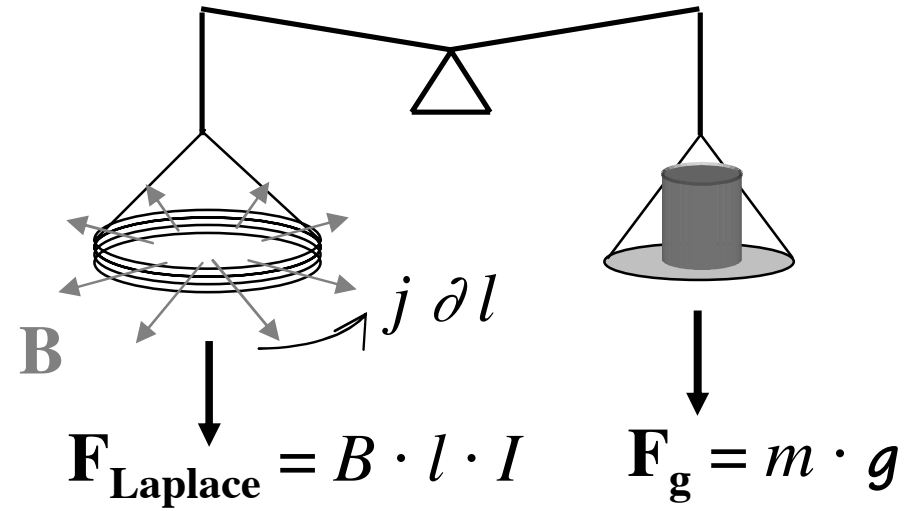
Le gravimètre atomique du LNE-SYRTE

La gravi-gradio-métrie

IPK Stabilité



Balance du watt

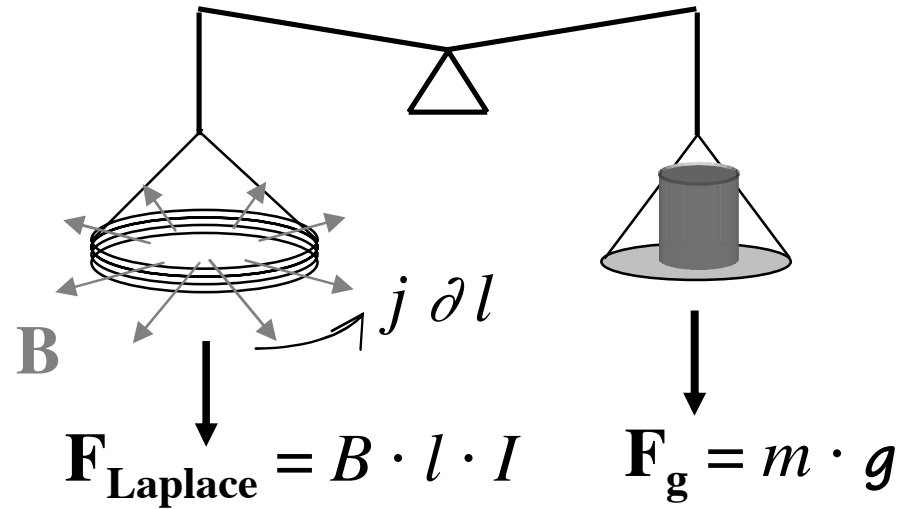


$$F_z = m \cdot g = B l i$$

Geometric factor

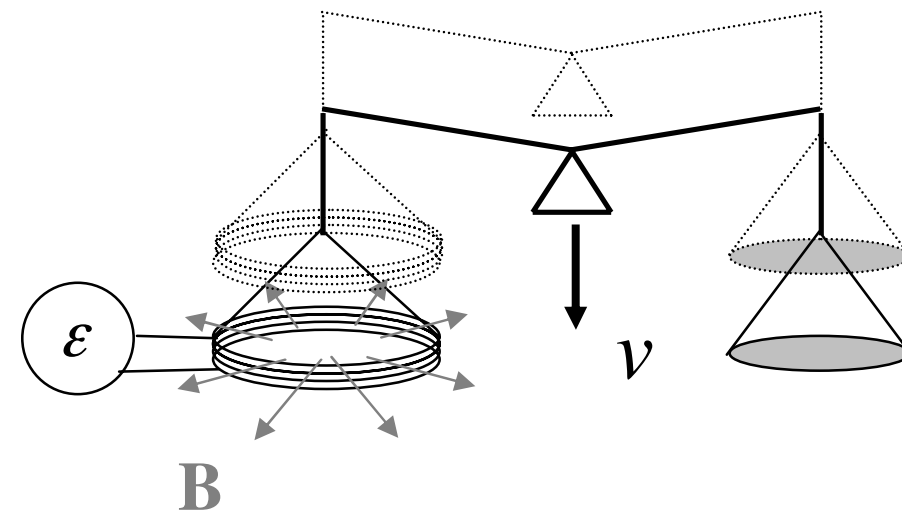
Balance du watt ou de Kibble

Etape statique



$$F_z = mg = Bli$$

Etape dynamique

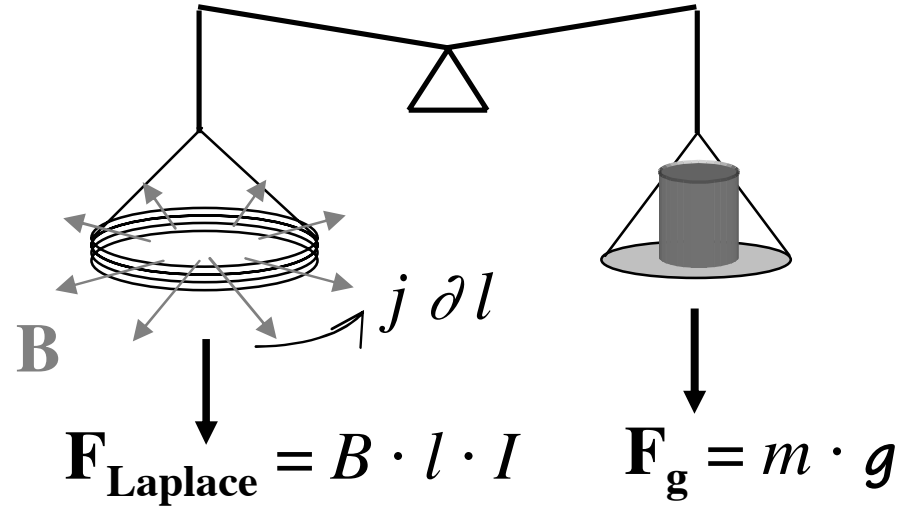


$$\epsilon = -Blv$$

$$mgv = \epsilon i = \epsilon V / R$$

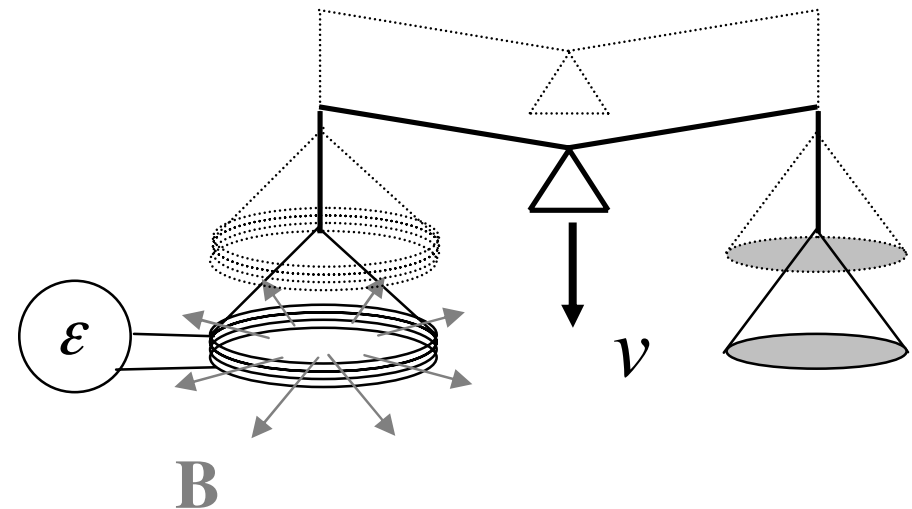
Balance du watt ou de Kibble

Etape statique



$$F_z = mg = Bli$$

Etape dynamique



$$\epsilon = -Blv$$

$$mgv = \epsilon i = \epsilon V / R$$

$$mgv = \frac{A}{K_J^2 R_K}$$

$$A = \frac{n_1 f_1 n_2 f_2 i}{k}$$

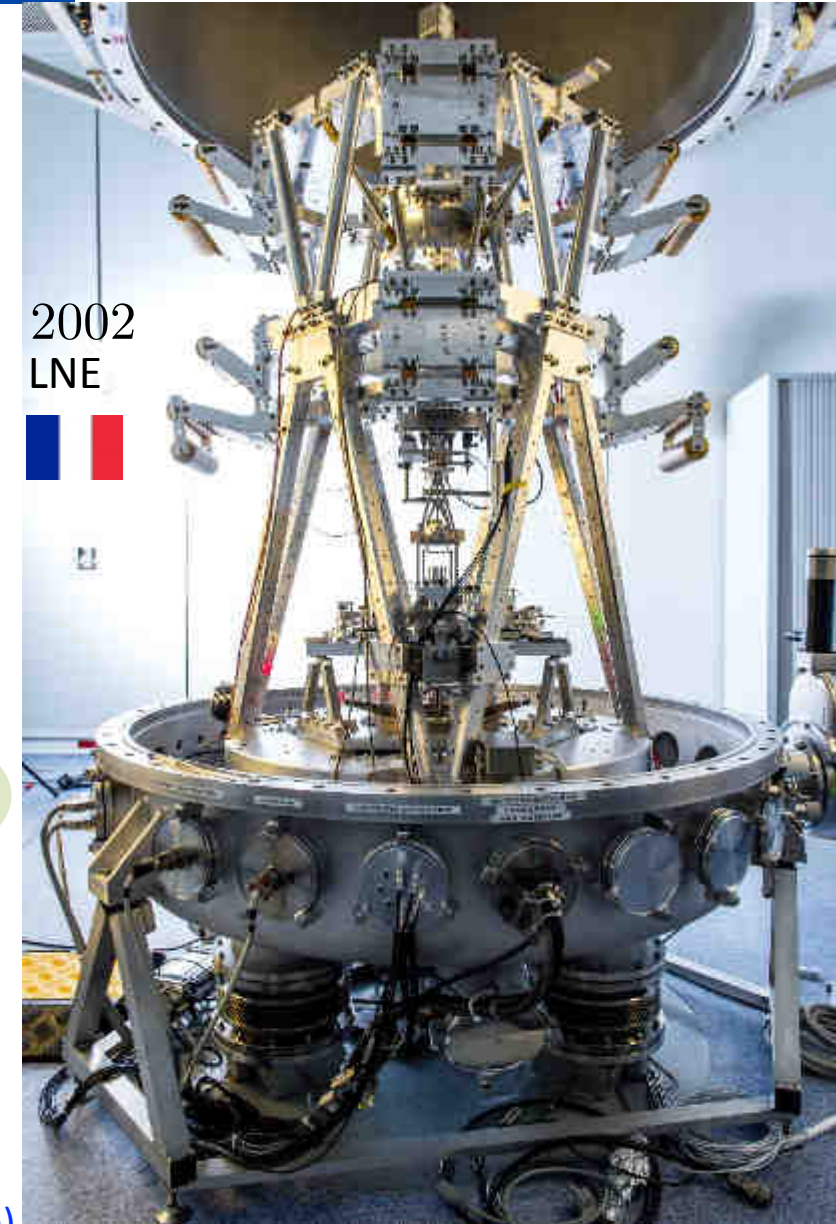
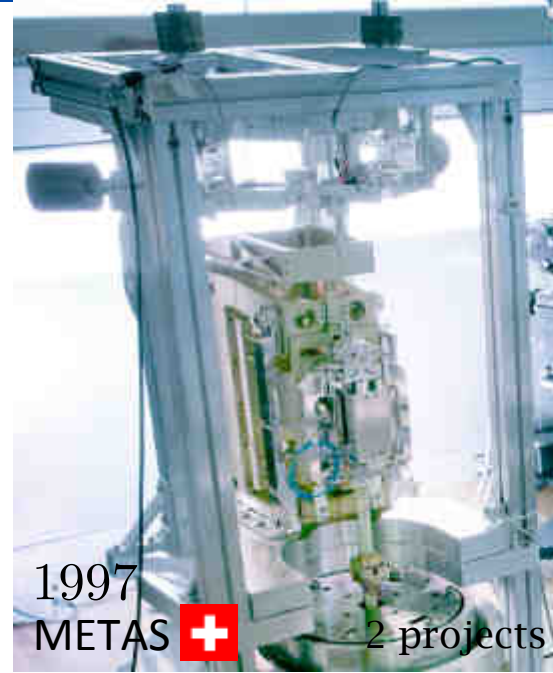
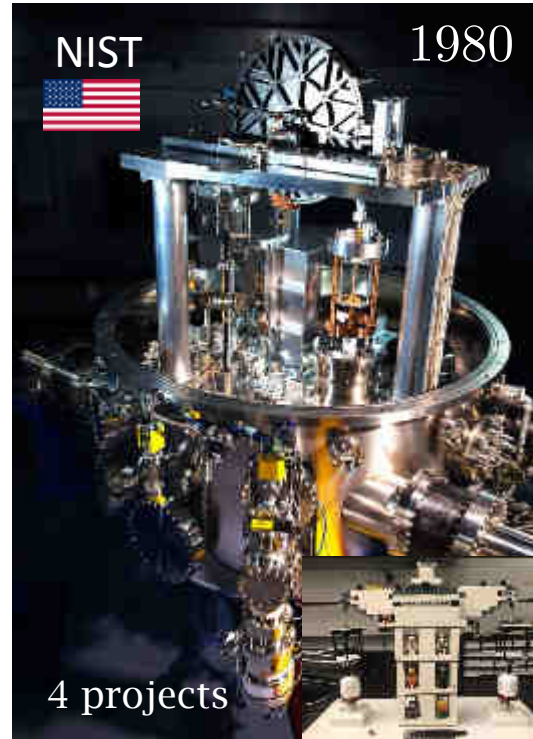
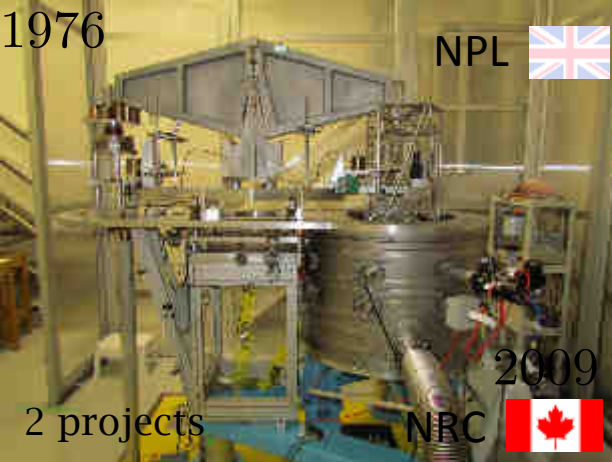
$$\frac{m}{h} = \frac{A}{4gv}$$

Besoin d'une mesure de g , **exacte**

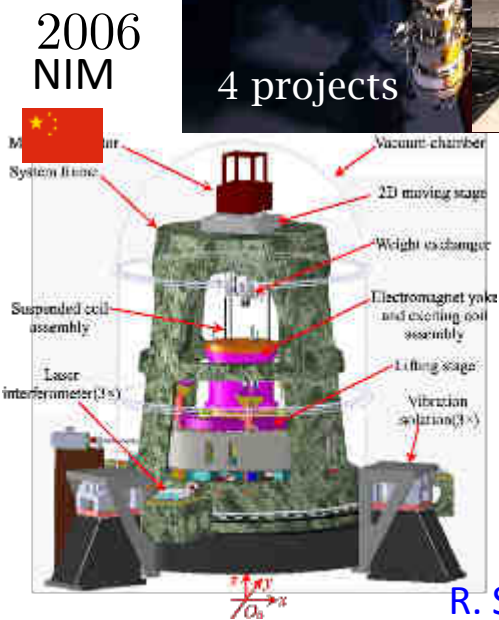
1962: Josephson effect
 $V = nf/K_J$ $K_J = 2e/h$

1980: Quantum Hall effect
 $R_H(i) = R_K/i$ $R_K = h/e^2$

Projets de balance de Kibble



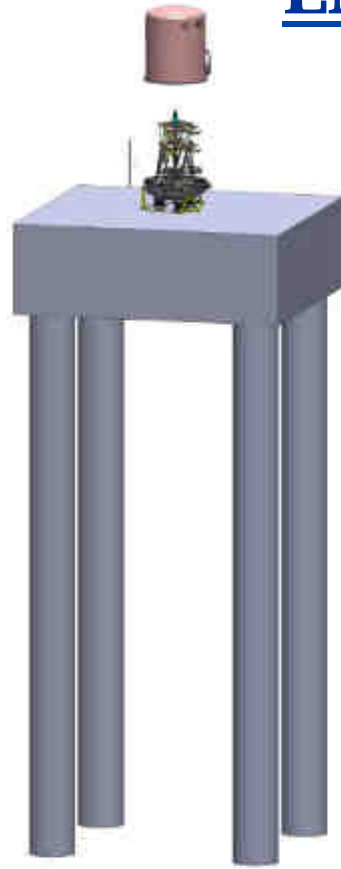
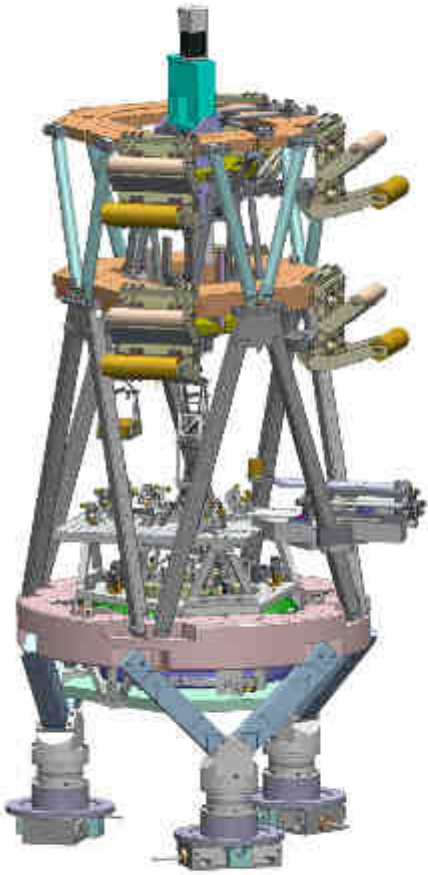
Beaucoup de différences (tailles, aimants, gravi...)



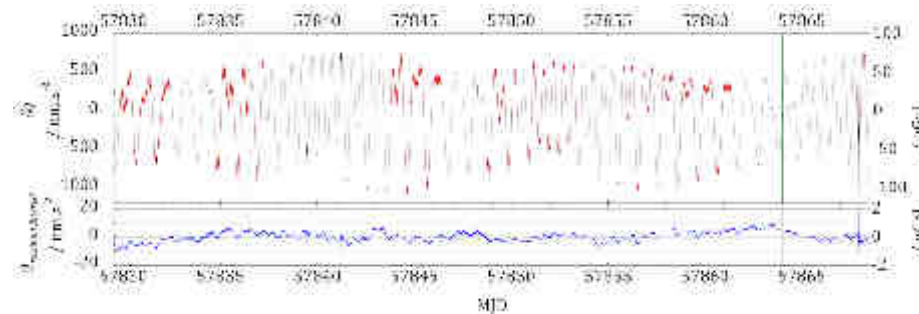
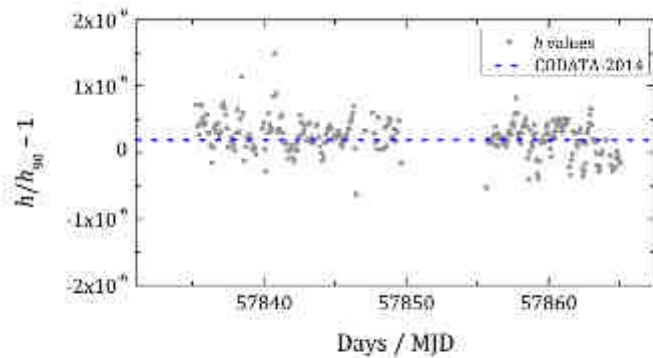
MSL, VNIIM...

R. Steiner Rep. Prog. Phys. 76 (2013) 016101 (46pp)

LNE KB 2017



1T
 Diamètre bobine: 266 mm
 Mouvement bobine : 80 mm
 Vitesse bobine: 2 mm/s
 Masse : 500g

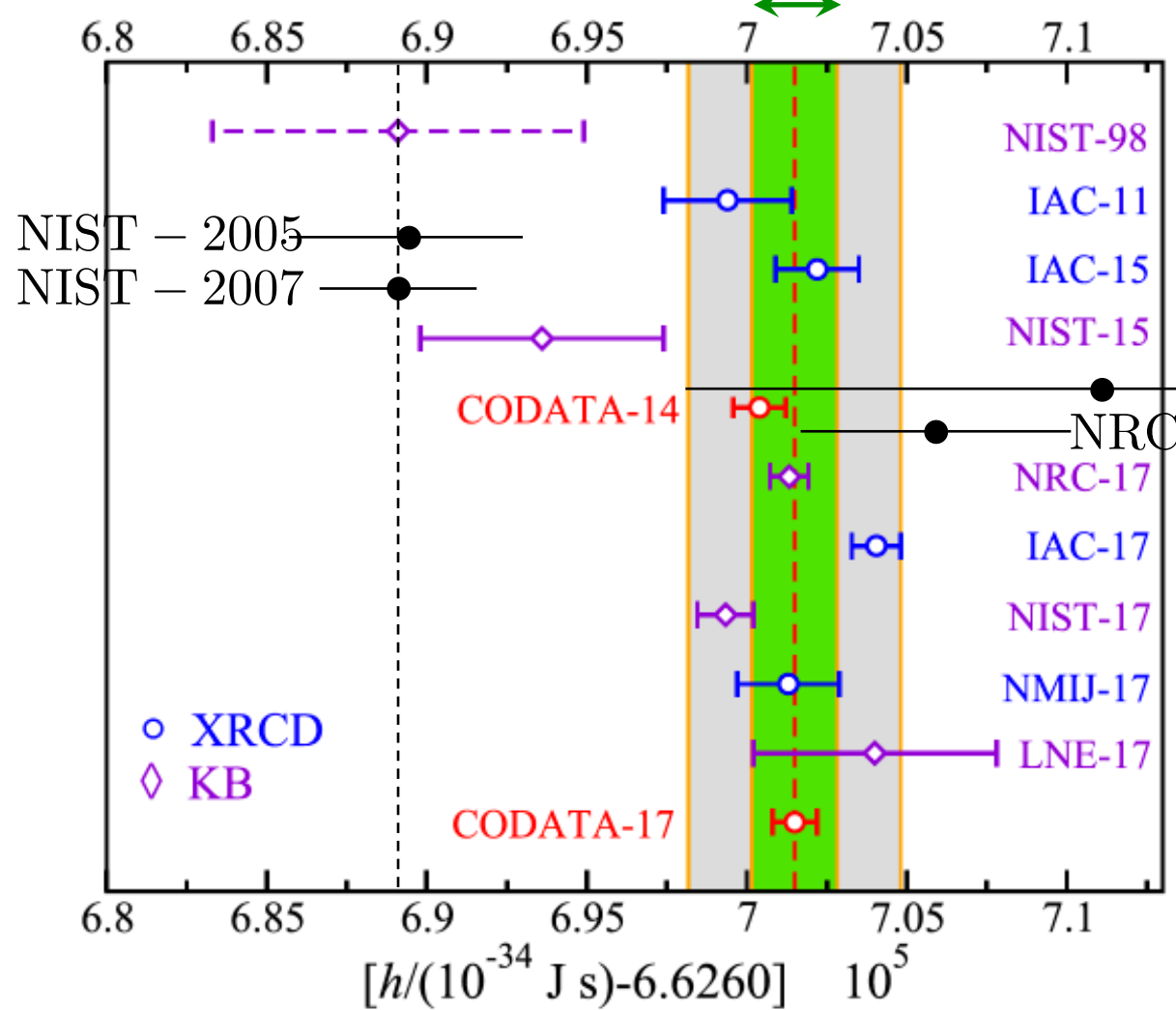


Uncertainty budget for h measurements		(10 ⁻⁹)
Type A		20
Type B		
	Voltage measurements U^l and U^h	11
	Resistance R	6
	500 g iridium mass	13
	Absolute gravity value g	5
	Velocity v	46
	Parasitic wall ratio term	17
	Force comparator contribution	8
	Other contributions (Abbe error, polynomial order, coil position, mathematical, reproducibility...)	10
Combined relative uncertainty		57

Résultats

P. J. Mohr et al (2018) Metrologia
D. B. Newell et al (2018) Metrologia

$\pm 5 \times 10^{-8}$
 $\pm 2 \times 10^{-8}$



5 labos/consortium publiés 8 valeurs mesurées de h
qui ont été prises en compte

IAC - 11 $\rightarrow 3.0 \times 10^{-8}$

IAC - 15 $\rightarrow 2.0 \times 10^{-8}$

NIST - 15 $\rightarrow 5.7 \times 10^{-8}$

NPL - 2012

NRC - 2012

NRC - 17 $\rightarrow 0.9 \times 10^{-8}$

IAC - 17 $\rightarrow 1.2 \times 10^{-8}$

NIST - 17 $\rightarrow 1.3 \times 10^{-8}$

NMIJ - 17 $\rightarrow 2.4 \times 10^{-8}$

LNE - 17 $\rightarrow 5.7 \times 10^{-8}$

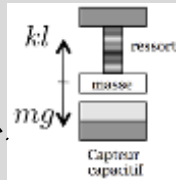
CODATA - 17 $\rightarrow h = 6.626\,070\,150(69) \times 10^{-34} \text{ J.s}$

For SI rev $\rightarrow h = 6.626\,070\,15 \times 10^{-34} \text{ J.s}$

Gravimétrie pour la balance de Kibble

g transfert

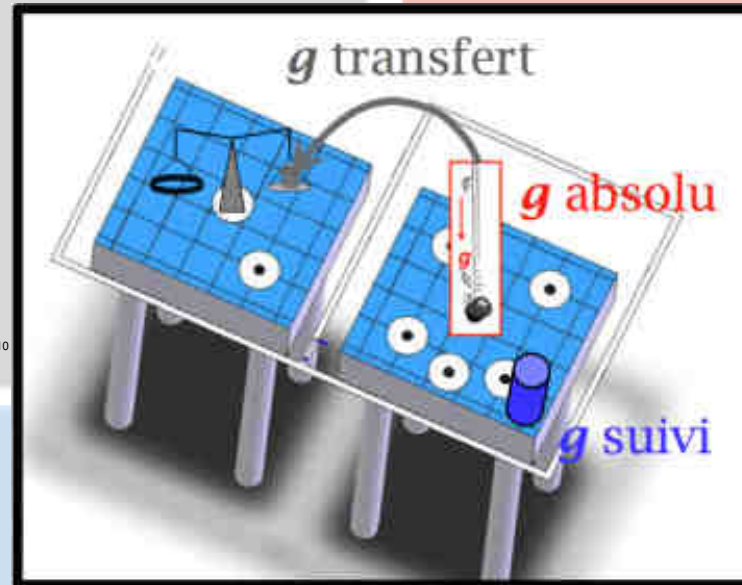
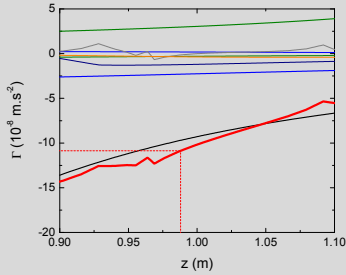
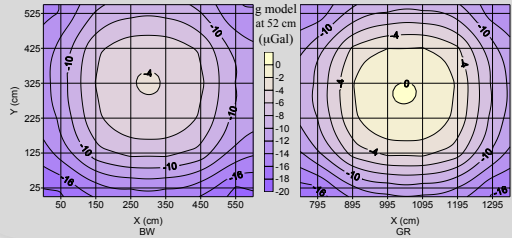
- Mesures relatives, cartographie 3D



Gravimètre relatif
 Scintrex CG5
 Grosse dérive
 ($4 \cdot 10^{-7} g/\text{jour}$)!
 6Hz

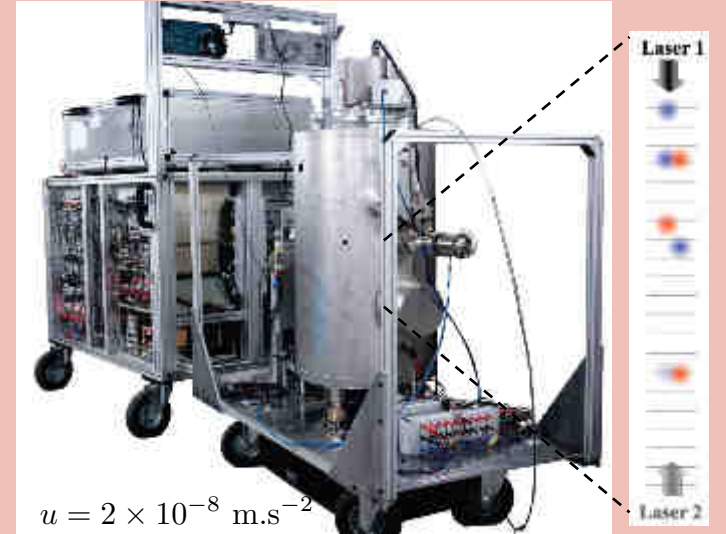


- Modélisation 3D (labo et BK)



g absolu (et continu)

Observation de la chute libre d'un corps: des atomes froids de ^{87}Rb par Interférométrie Atomique.



$$u = 2 \times 10^{-8} \text{ m.s}^{-2}$$

Développement d'une référence nationale, le **CAG Absolu, déplaçable, continu**

3Hz

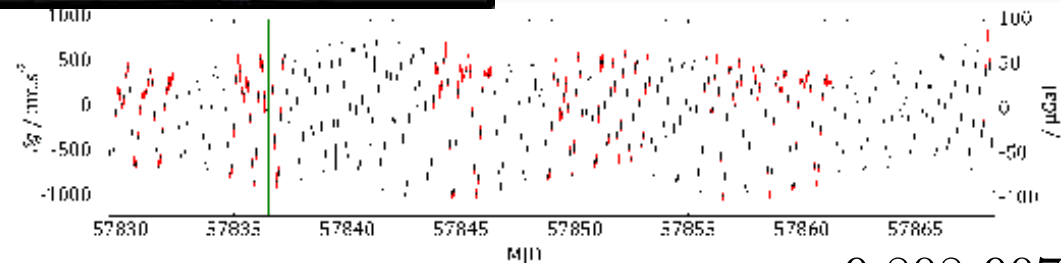
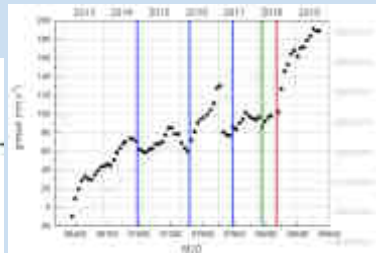
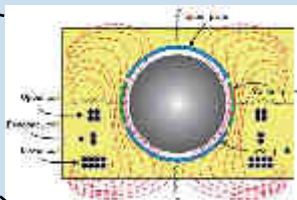
Participation aux comparaisons internationales (CIPM KC)

Transfert industriel, commercialisation



g suivi continu

Gravimètre supraconducteur relatif GWR iGrav
 Faible dérive ($4 \cdot 10^{-9} g/\text{an}$)
 12Hz
 Non déplaçable



$$g = 9.808\,907\,45(2) \text{ m.s}^{-2}$$

Sommaire

Notre participation dans ce changement

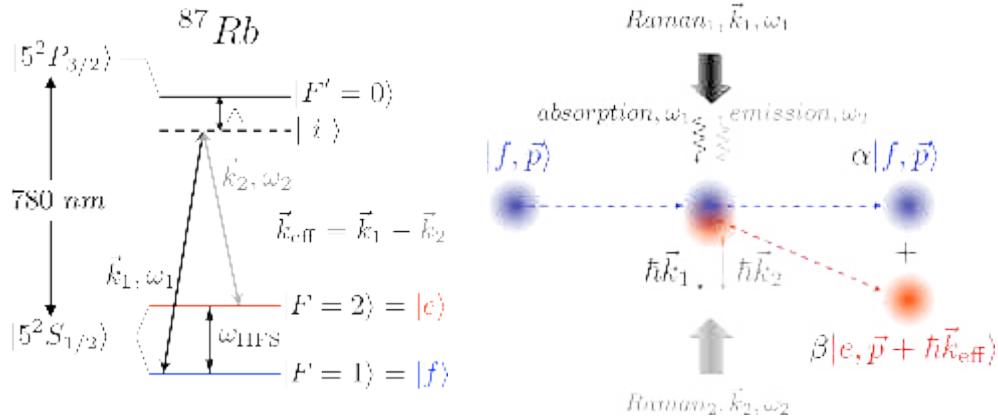
Le gravimètre atomique du LNE-SYRTE

La gravi-gradio-métrie

Atom interferometer

Stimulated Raman transitions

3 level atom



Two photon transition coupling $|f\rangle$ and $|e\rangle$

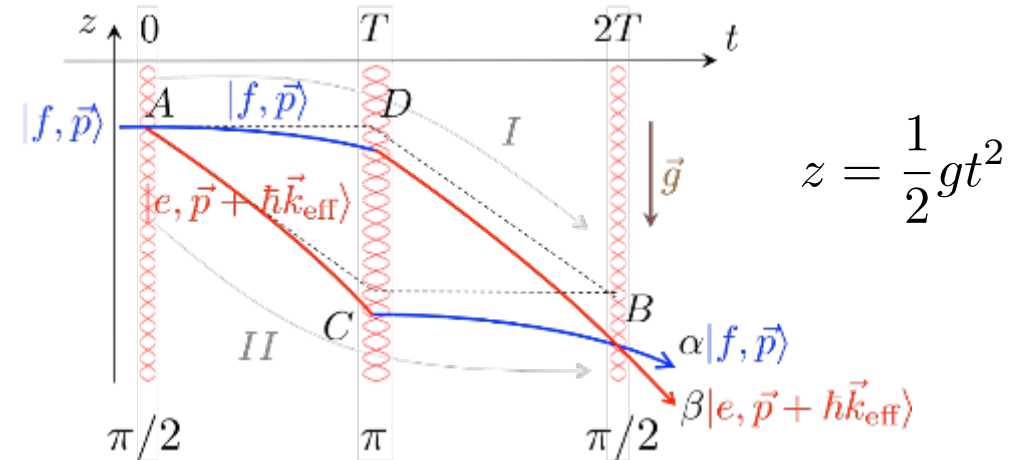
$$\phi(t) = \omega_{\text{eff}}t - \vec{k}_{\text{eff}}\vec{z}(t) + \varphi_{\text{eff}}(t)$$

Interest of Raman transitions:

Bijection internal - external state

Consequence : detection on internal state

Sampling of the positions at the 3 pulses



$$P_{|\vec{p}\rangle \rightarrow |\vec{p} + \hbar\vec{k}_{\text{eff}}\rangle} = \frac{1}{2}(1 - C \cos \Delta\Phi)$$

$$\begin{aligned} \Delta\Phi &= \Phi_{II} - \Phi_I \\ &= (\phi_A - \phi_C) - (\phi_D - \phi_B) \\ &= \phi(0) - 2\phi(T) + \phi(2T) \end{aligned}$$

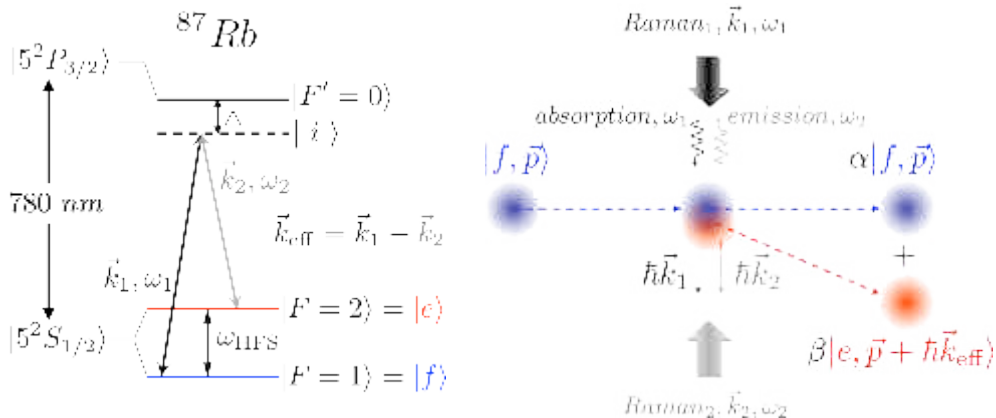
$$\Delta\Phi = -\vec{k}_{\text{eff}}\vec{g}T^2$$

↓
Scales as T^2 , benefits of cold atoms

Atom interferometer

Stimulated Raman transitions

3 level atom



Two photon transition coupling $|f\rangle$ and $|e\rangle$

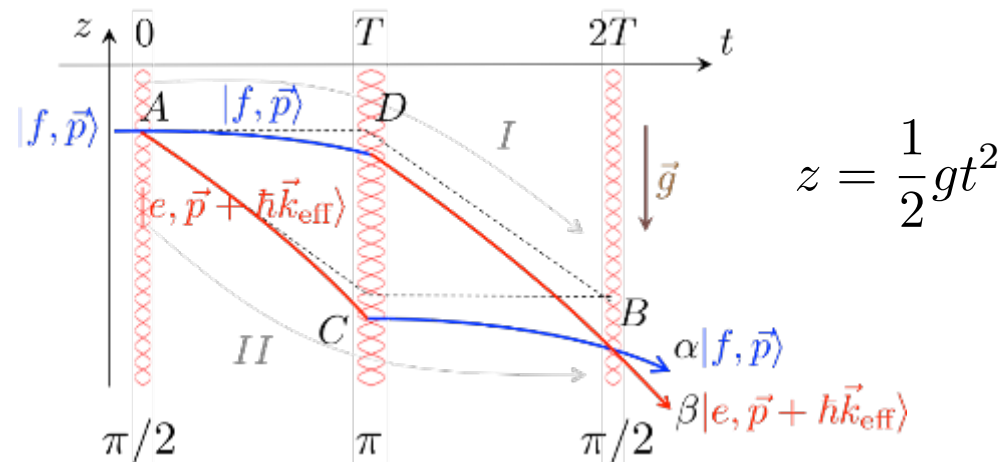
$$\phi(t) = \omega_{\text{eff}}t - \vec{k}_{\text{eff}}\vec{z}(t) + \varphi_{\text{eff}}(t)$$

Interest of Raman transitions:

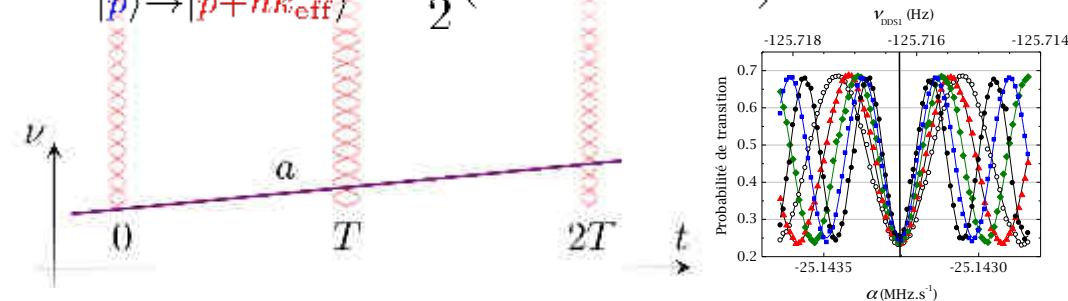
Bijection internal - external state

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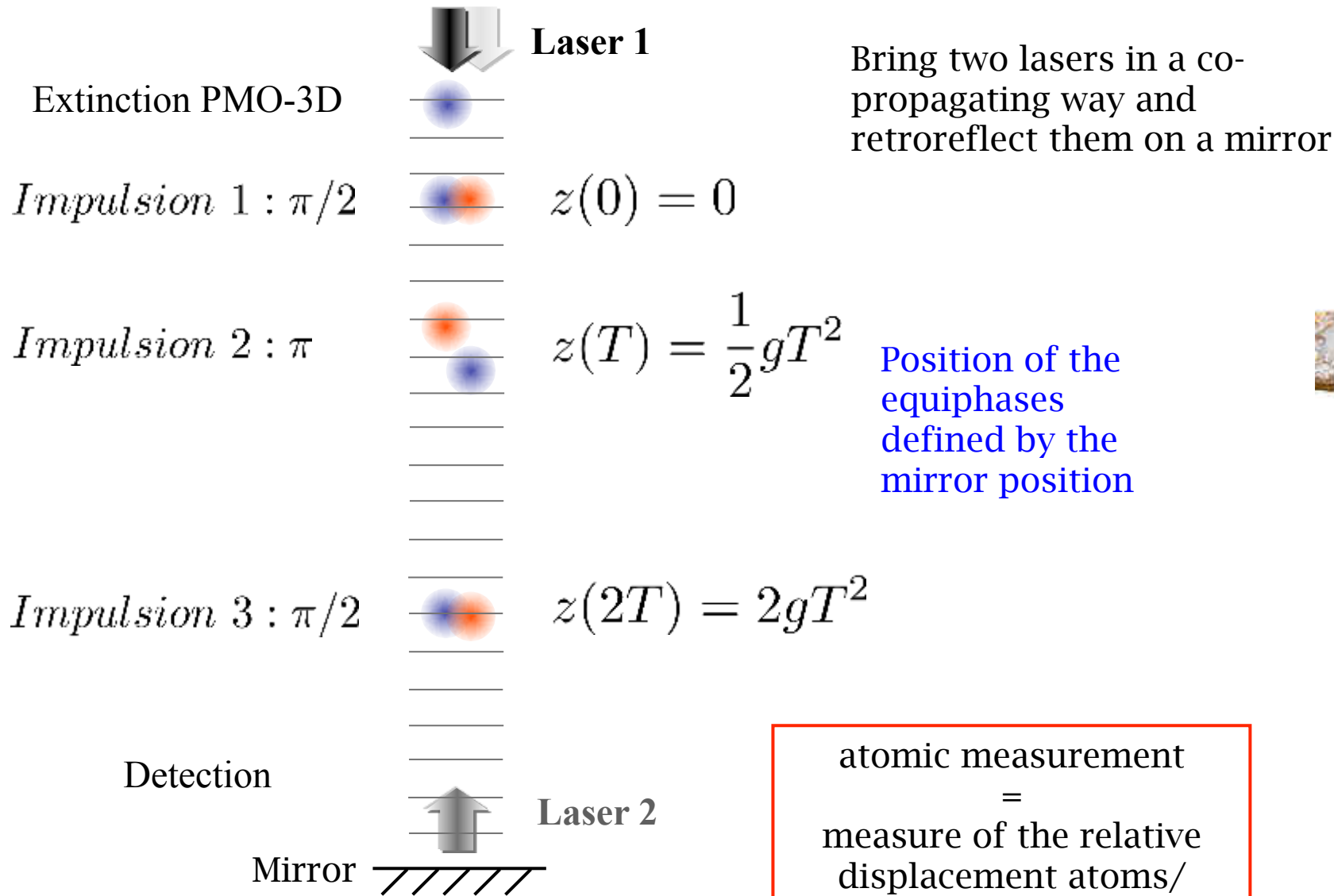


$$\Delta\Phi = -\vec{k}_{\text{eff}}\vec{g}T^2 + aT^2$$

$$g = a/k_{\text{eff}}$$

Scales as T^2 , benefits of cold atoms

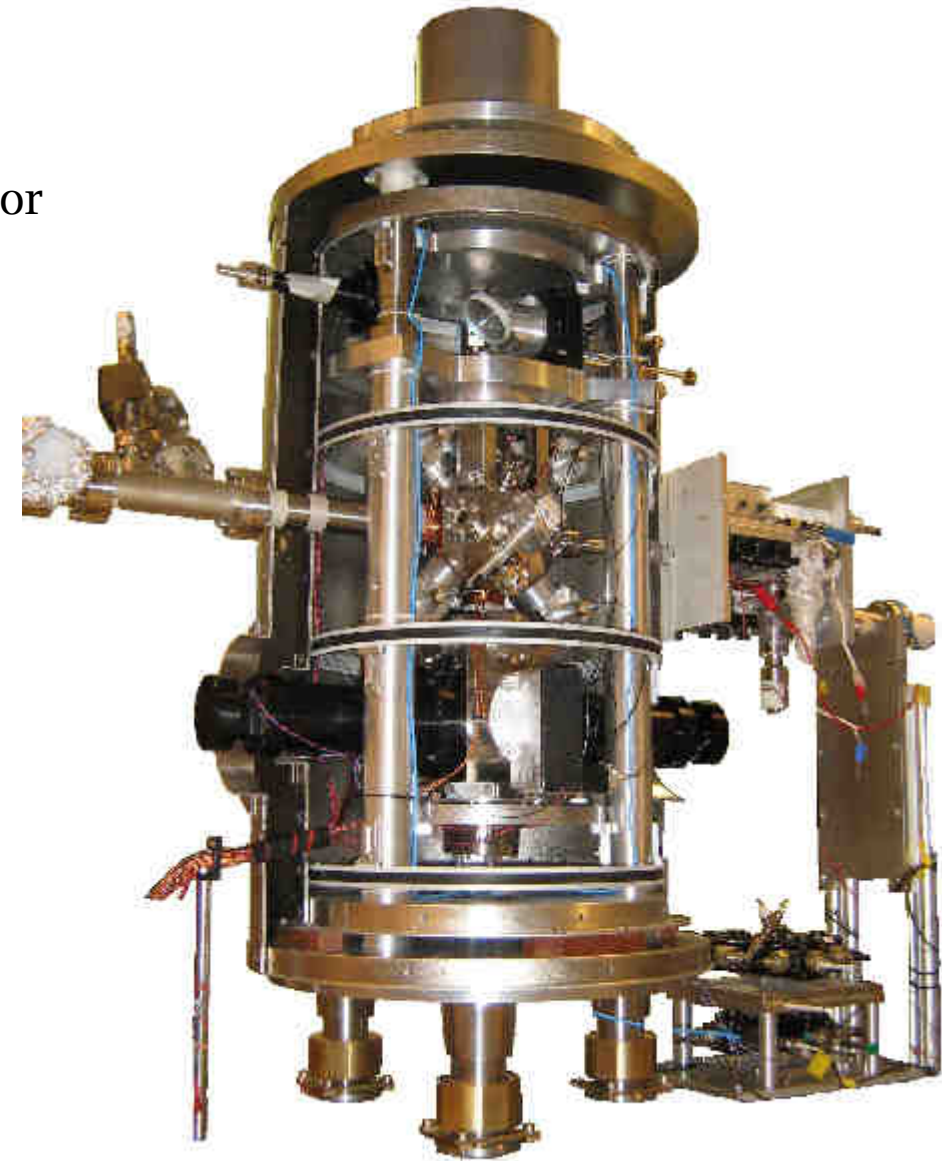
Cold Atom Gravimeter (CAG) sequence



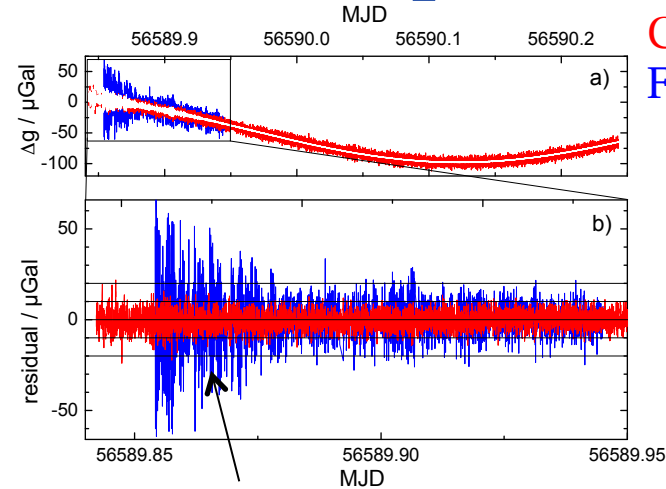
Bring two lasers in a co-propagating way and retroreflect them on a mirror

Position of the equiphases defined by the mirror position

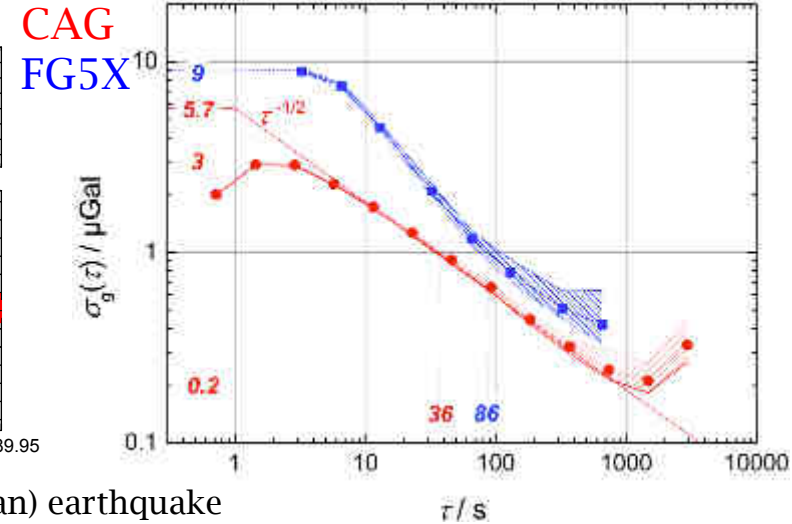
atomic measurement
=
measure of the relative displacement atoms/
mirror



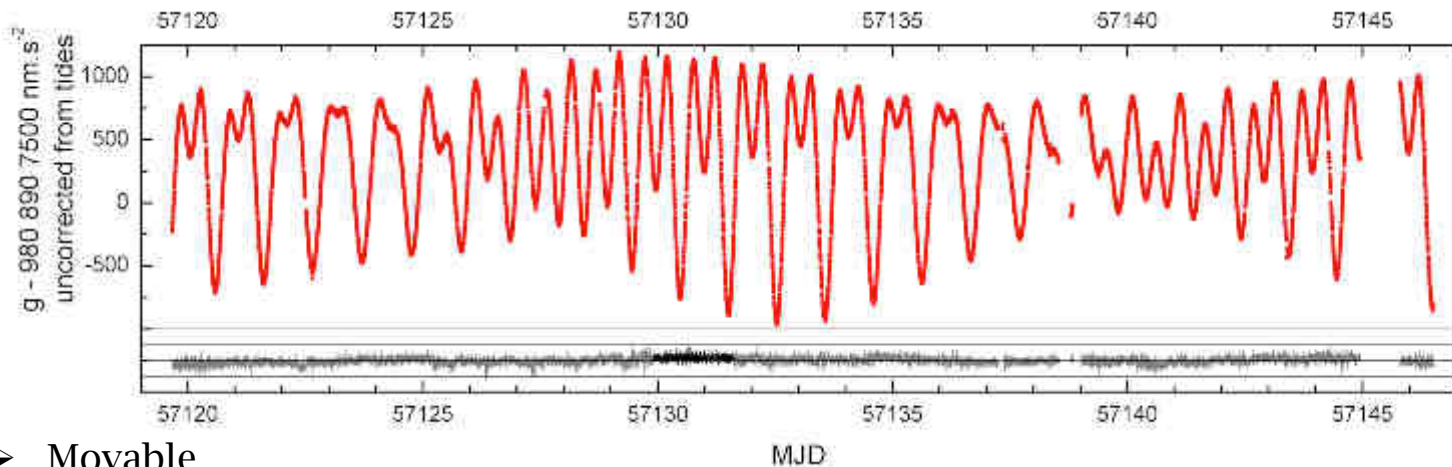
CAG performances & capabilities



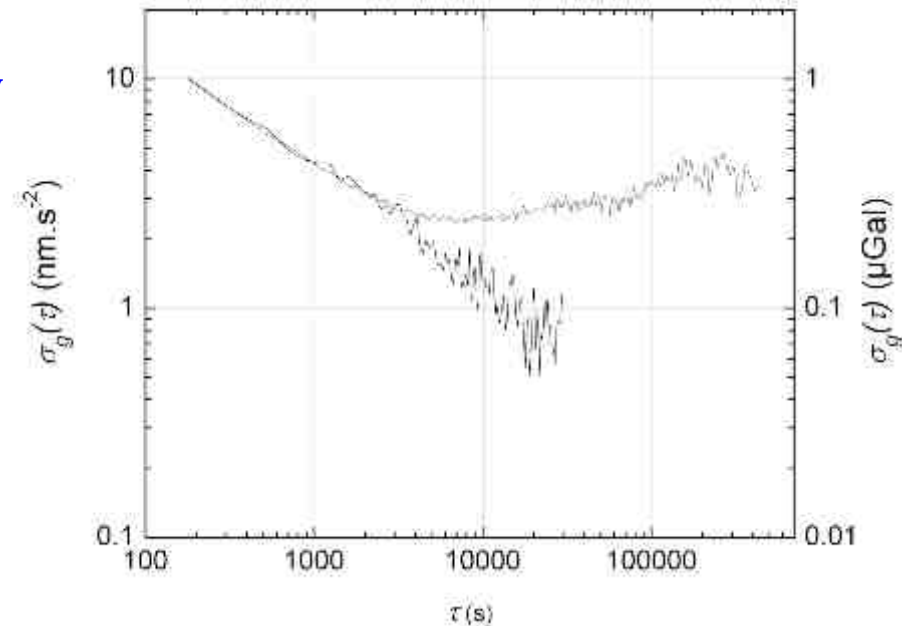
Excess noise due (end of an) earthquake
Better immunity of the CAG



P. Gillot et al., Metrologia 51 (2014)



CAG
 iGrav

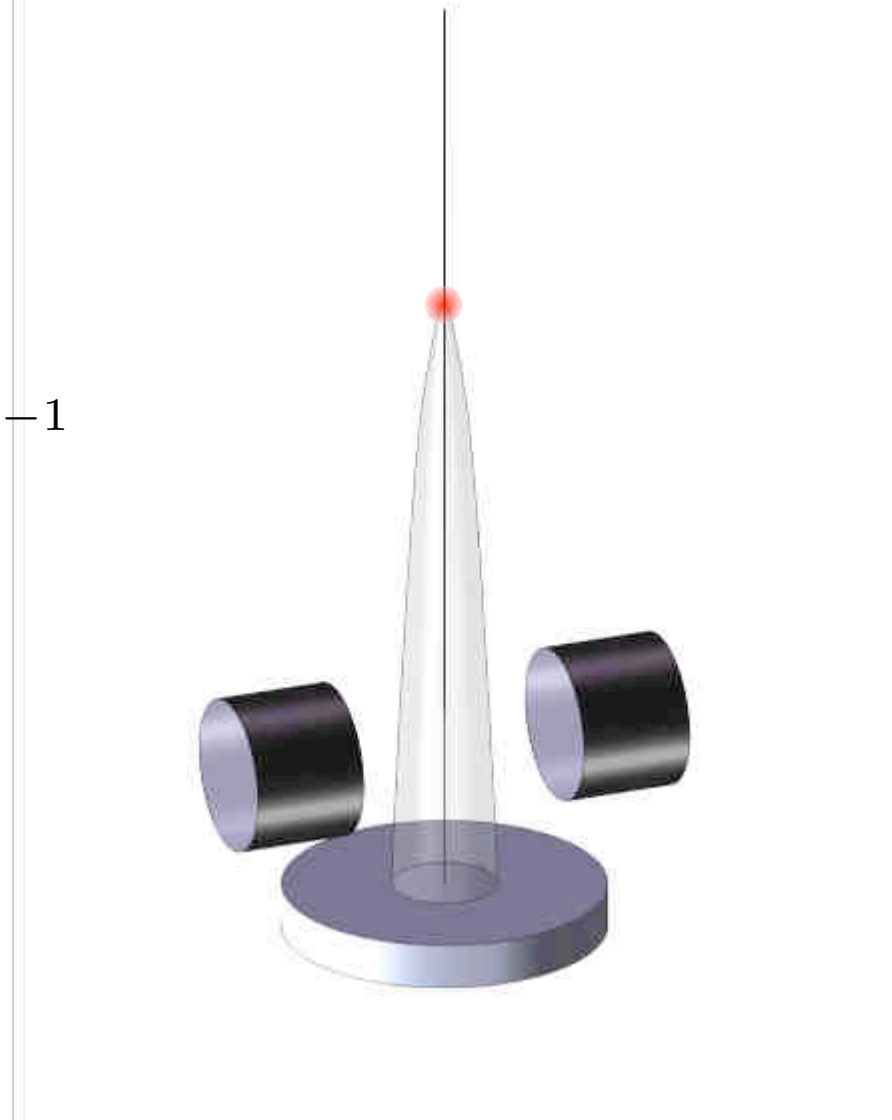


Accuracy : 4.3 μGal

- Movable
- Comparison with other technologies
- Participation to international Key Comparisons
- **Continuous accurate measurement**
- **Industrial transfer**

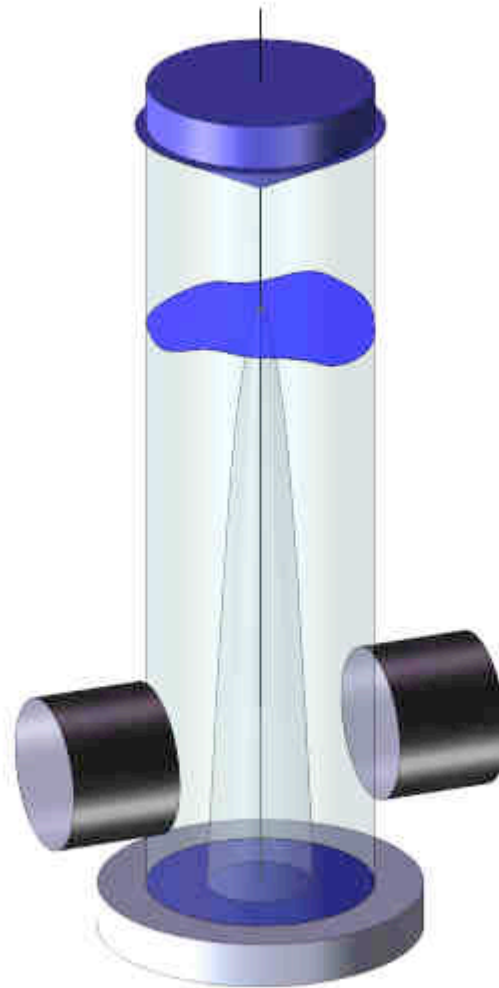
Free fall and cloud expansion

Cold Atoms
 $T_{emp} \sim 2\mu K$
 $\sigma_v \sim 14 \text{ mm.s}^{-1}$



1st Raman

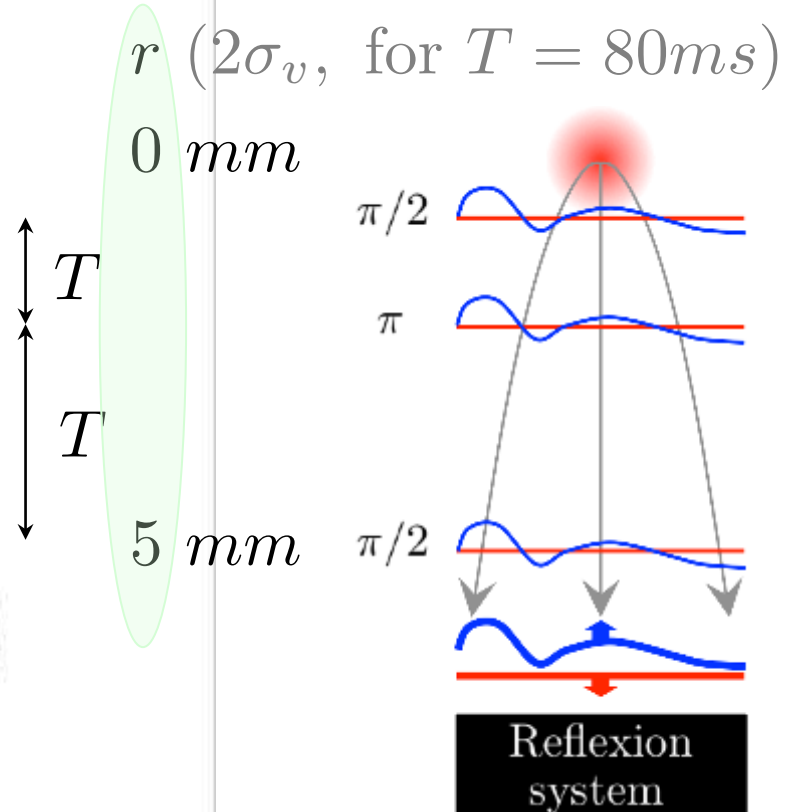
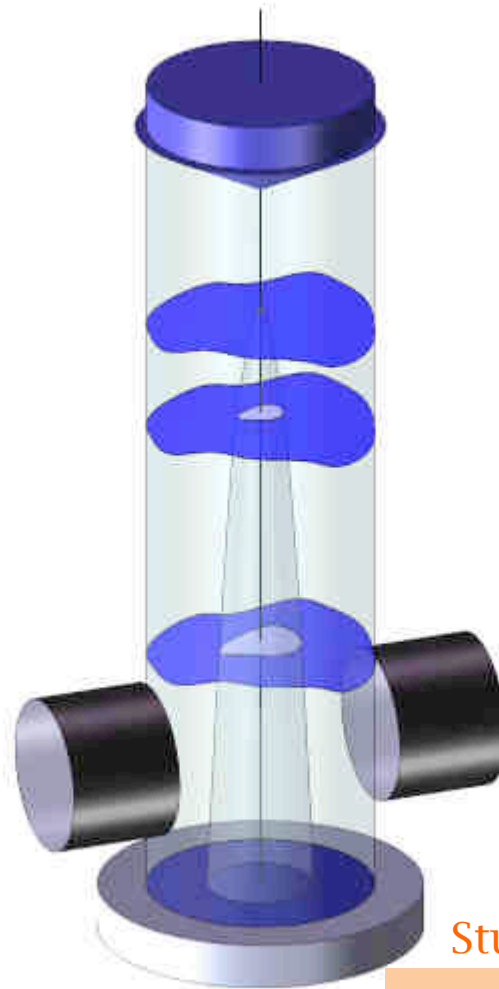
Laser beam



Reflexion
system

Wavefront distortion and cloud expansion

Cold Atoms
 $T_{emp} \sim 2\mu K$
 $\sigma_v \sim 14 \text{ mm.s}^{-1}$



Study case: curvature

$$\phi \sim Kr^2 \quad \Delta\Phi = 2K\sigma_v^2 T^2 = \frac{k_{eff}}{R} \frac{k_B T_{emp}}{m_{Rb}} T^2$$

for $\Delta g = 10^{-9}g$, $R = 20km$, $\lambda/300 \text{ PV}$ ($2r = 1cm$)

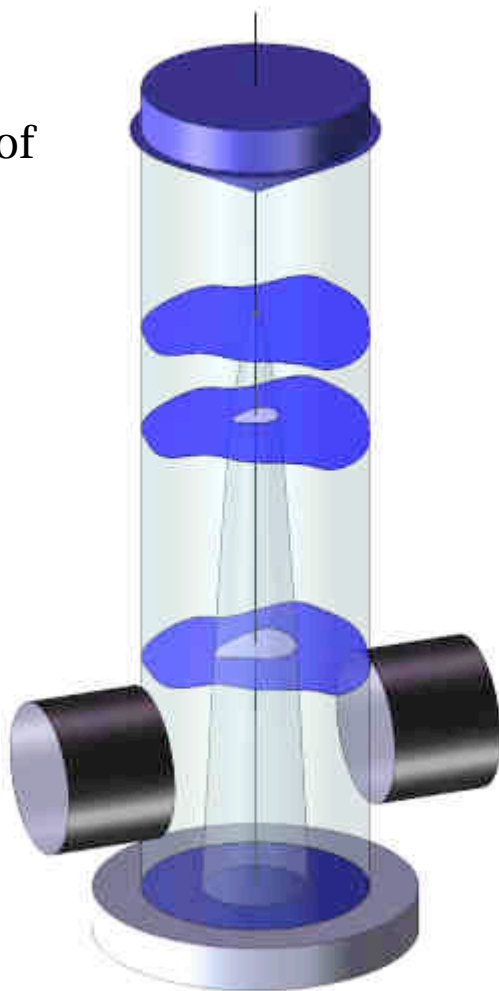
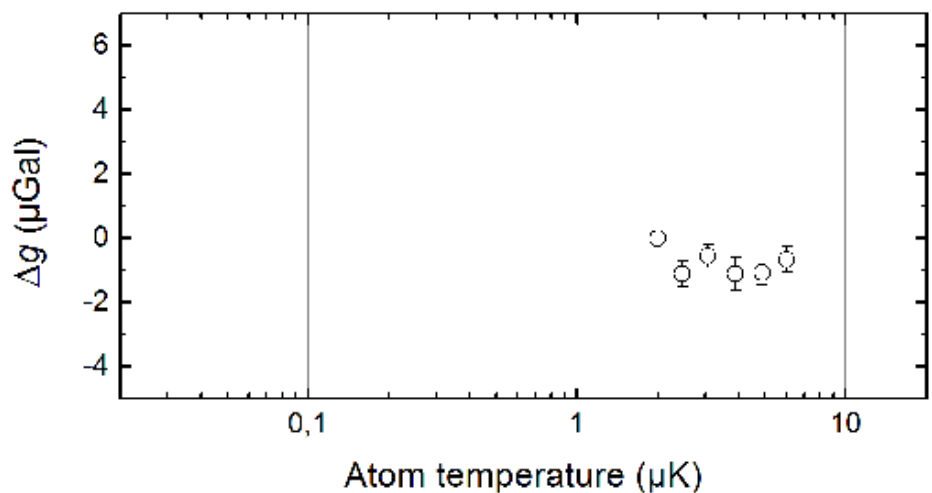
Following an atomic trajectory the total phase coming from wavefront imprinted at each pulse is non zero due to the ballistic expansion of the atomic cloud.

Wavefront Bias determination

Previous determination:

Increase the temperature to modify the effect of **wavefront aberrations**

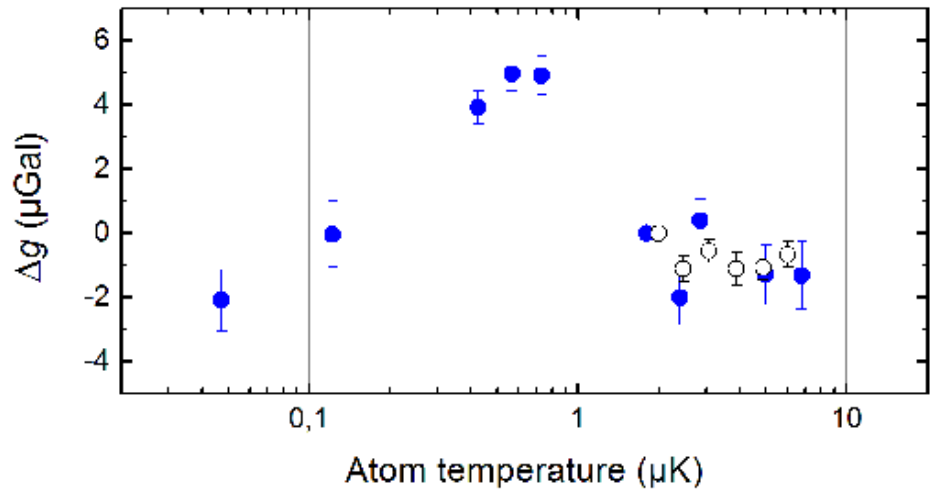
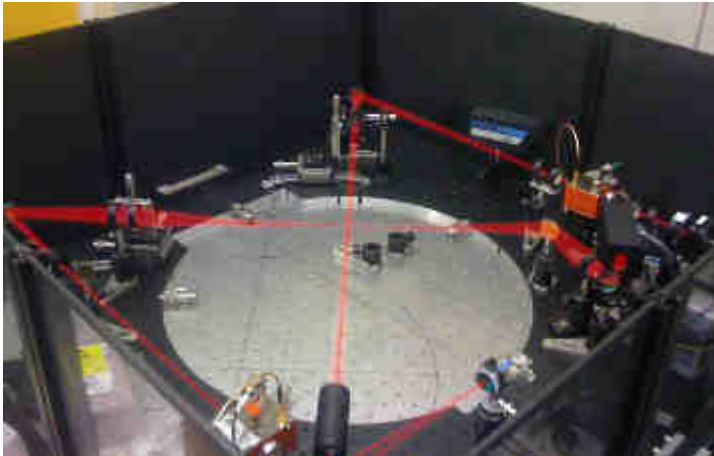
A. Louchet-Chauvet et al New J. Phys. 13, 065025 (2011)



$$\Delta g = (0 \pm 4) \mu Gal$$

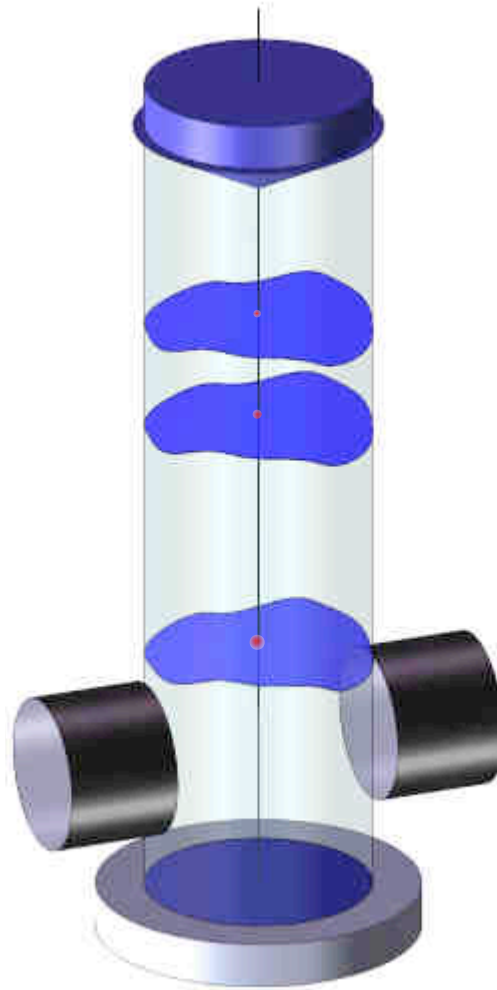
Effect	Bias μGal	u μGal
Alignments	0.3	0.5
Frequency reference	0.5	<0.1
RF phase shift	0.0	<0.1
<i>v_{gg}</i>	-13.4	<0.1
Self gravity effect	-2.1	0.1
Coriolis	-5.3	0.8
Wavefront aberrations	0.0	4.0
LS1	0.0	<0.1
Zeeman	0.0	<0.1
LS2	-3.6	0.8
Detection offset	0.0	0.5
Optical power	0.0	0.5
Cloud indice	0.4	<0.1
Cold collisions	<0.1	<0.1
CPT	0.0	<0.1
Raman α LS	0.3	<0.1
Finite Speed of Light	0.0	<0.1
TOTAL	-22.9	4.3

Implement Ultracold Source on CAG



Differential gravity measurements as a function of the atomic temperature from 50nK to 7μK

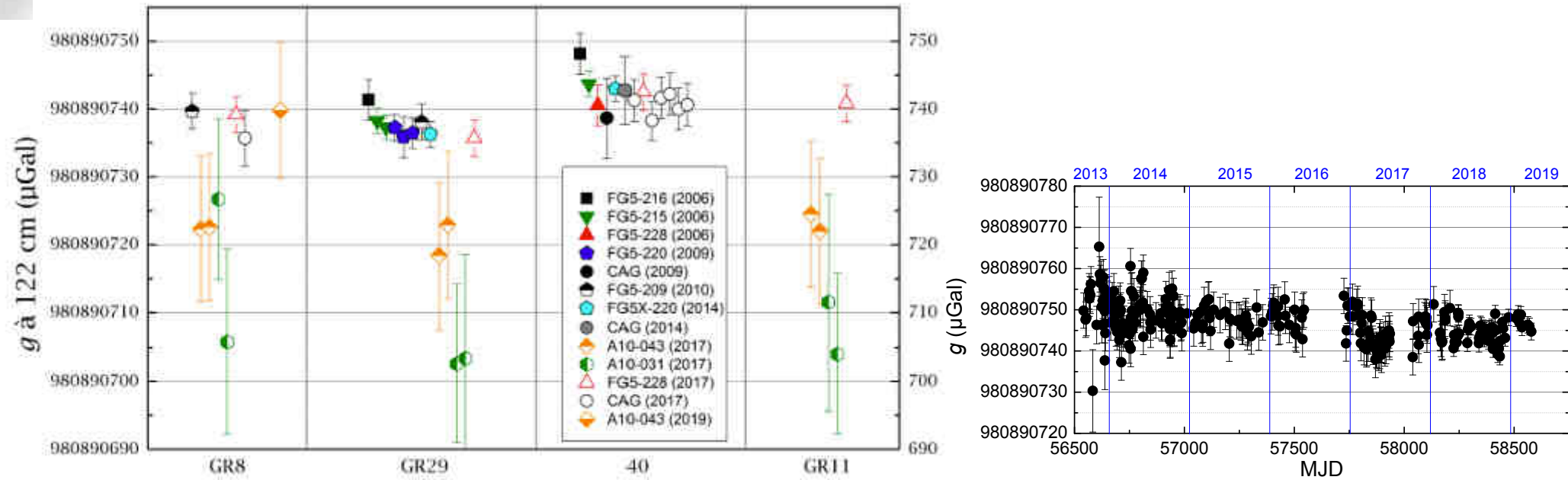
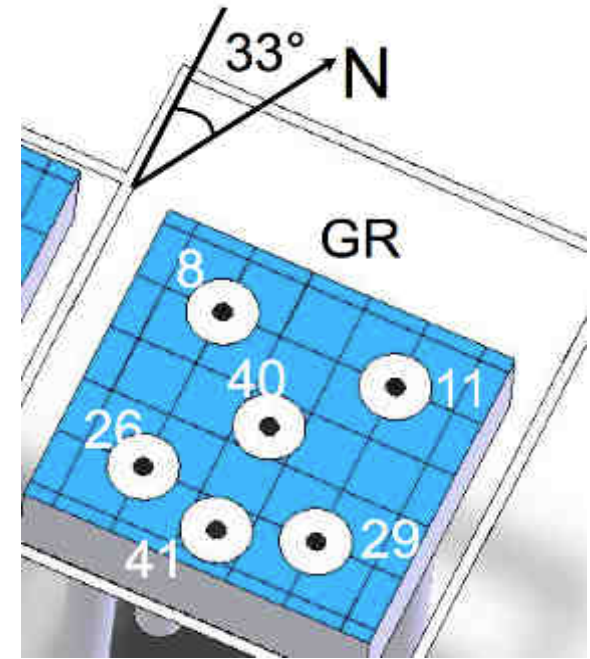
R. Karcher et al., *New J. Phys.* **20** (2018) 113041



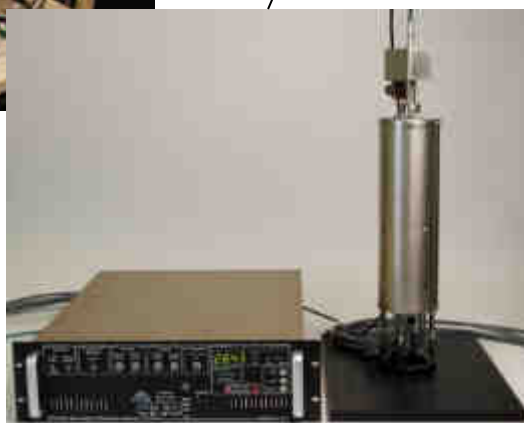
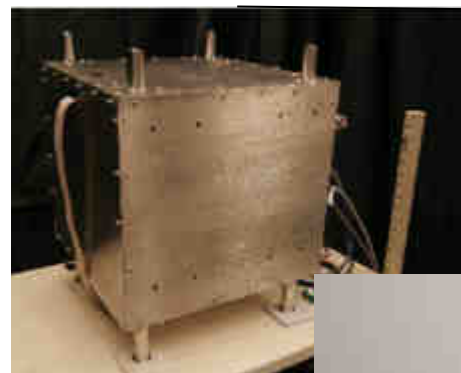
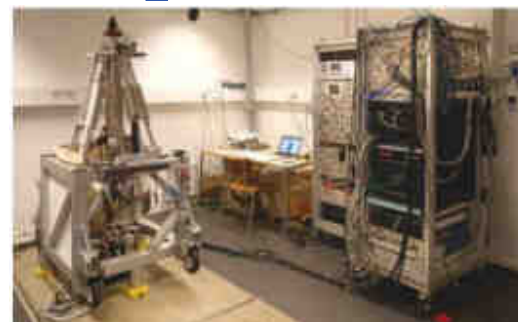
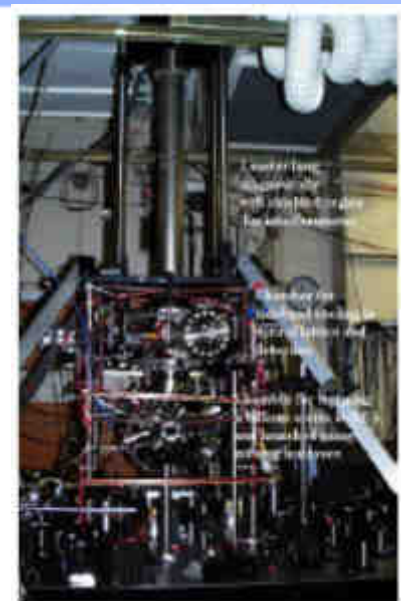
$$\Delta g = (-5.3 \pm 1.3) \mu Gal$$

Effect	Bias μGal	u μGal
Alignments	0.3	0.5
Frequency reference	0.5	<0.1
RF phase shift	0.0	<0.1
<i>v</i> gg	-13.4	<0.1
Self gravity effect	-2.1	0.1
Coriolis	-5.3	0.8
Wavefront aberrations	-5.6	1.3
LS1	0.0	<0.1
Zeeman	0.0	<0.1
LS2	-3.6	0.8
Detection offset	0.0	0.5
Optical power	0.0	0.5
Cloud indice	0.4	<0.1
Cold collisions	<0.1	<0.1
CPT	0.0	<0.1
Raman α LS	0.3	<0.1
Finite Speed of Light	0.0	<0.1
TOTAL	-28.5	2.0

National comparison



Domaine en pleine expansion



Comparison with other gravimeters

Type	Techno	Institut / Company	Name	ν_c Hz	u μGal	u_{wfab} μGal	σ_g μGal	τ s	Rq
Abs	FFCC	Micro-g Lacoste	FG5X	0.3 0.1	2.0 .	. .	\sim 1.0 1.0	\sim 100 \sim 700	dead time, wearing
Rel	Supra	GWR	iGrav	1.0	.	.	0.01	600	drift
Abs	Atom	SYRTE	CAG	2.8	2.0 . .	1.3 . .	5.7 1.0 0.06	1 36 20 000	T=80ms dropped atoms
Abs	Atom	HUB	GAIN	0.7	3.2 . .	2.2 . .	9.6 1.0 0.05	1 100 100 000	T=260ms launched atoms
Abs	Atom	HUST		0.5	5.0	4.2 1.0 0.3	1 18 200	T=300ms launched atoms
Abs	Atom	muquans	AQG		59.4 1.0 0.3	1 4000 200 000	dropped atoms
Abs	Atom	AOSense			.	.	.		
Abs	Atom	M Squared			.	.	.		

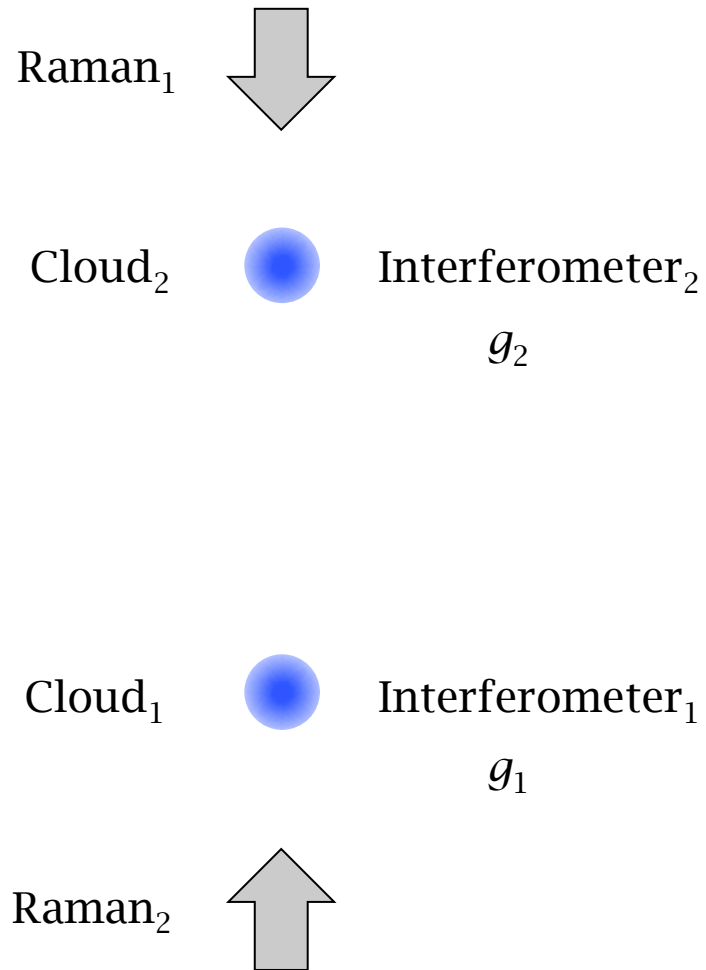
Sommaire

Notre participation dans ce changement

Le gravimètre atomique du LNE-SYRTE

La gravi-gradio-métrie

Quantum dual gravi-gradio meter



$g_1, g_2, g_1 - g_2$

- **Simultaneous** interferometers on two cold atom clouds with **common Raman lasers**
- **Differential measurement** allows for extracting the **acceleration difference** (and thus the Earth gravity gradient)
- **Suppression of common mode noise**, and in particular of the vibration noise
- Adapted for onboard measurements
- g and Δg : **resolve ambiguities** in determination of mass and position

How to increase the sensitivity ?

$$\Delta\Phi = \vec{k}_{\text{eff}} \vec{g} T^2$$

↑ ↑

Increase the scale factor

New tools

- High order Bragg diffraction LMBS with up to N photons
- Ultracold atoms
Fast generation on atom chip

Quantum dual gravi-gradio meter

- 2 ultracold Rb clouds obtained on 2 chips
- 2 clouds launched with elevator
- 2 Interferometers driven by LMTB

Targeted parameters

$$T_c = 2s \quad N_{\text{atoms}} = 5 \cdot 10^5$$
$$T_{\text{emp}} = 10 - 100nK$$
$$p = 100\hbar k \quad 2T = 0.5s$$

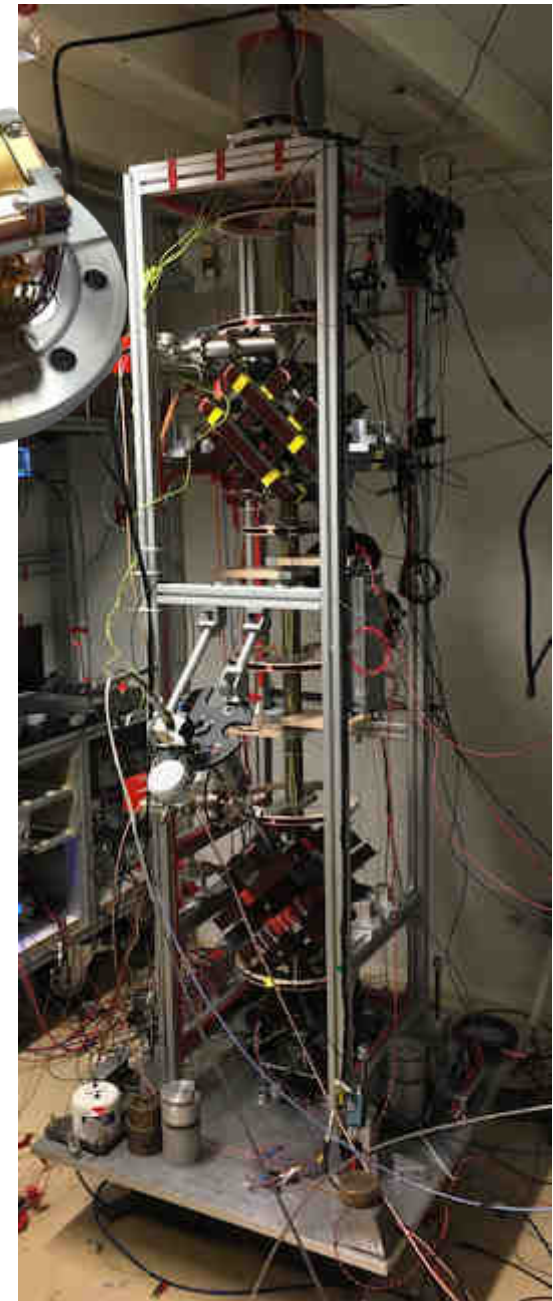
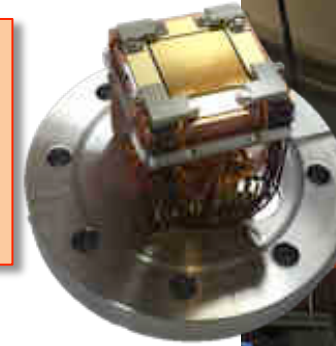
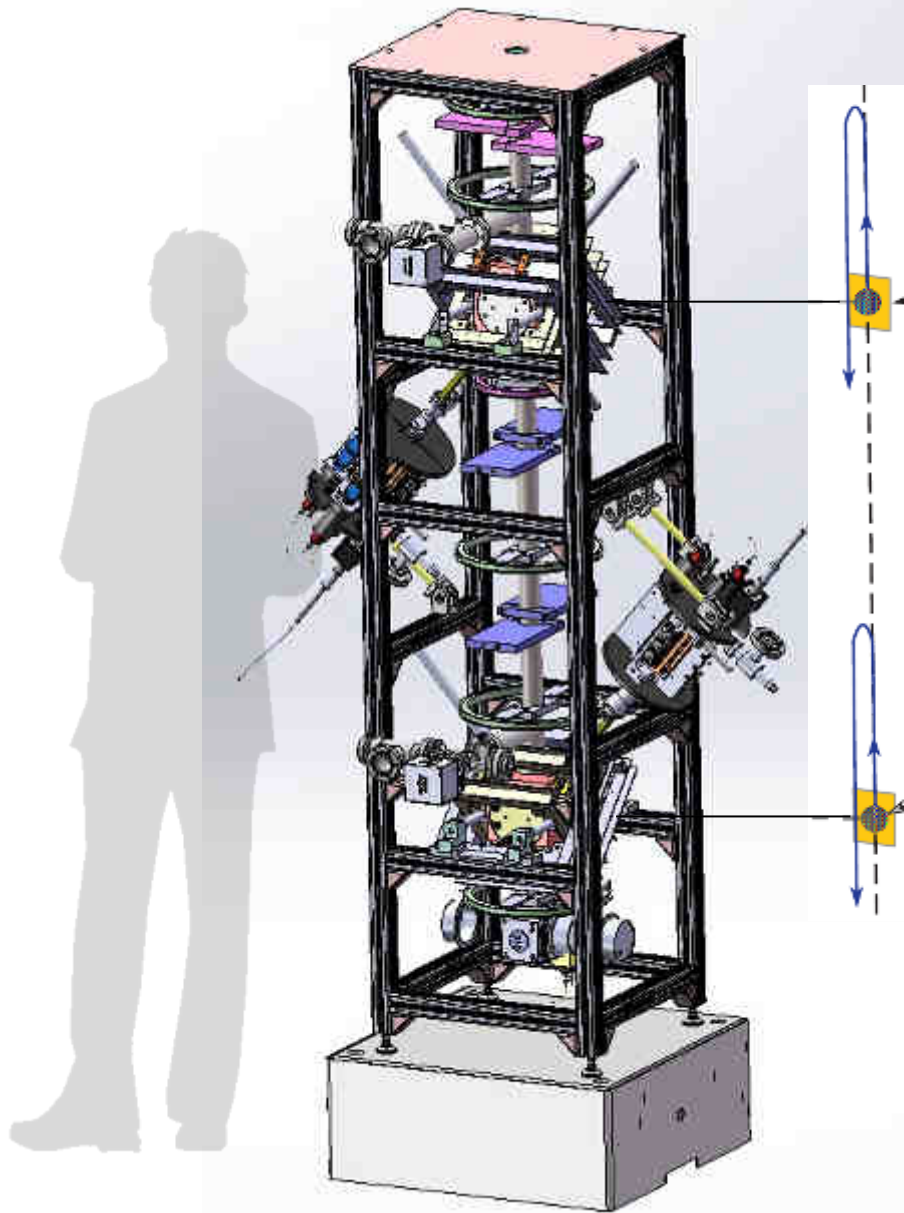
$$\sigma_g^1 = 9 \times 10^{-11} m \cdot s^{-2} \cdot Hz^{-1/2}$$

If limited by QNP

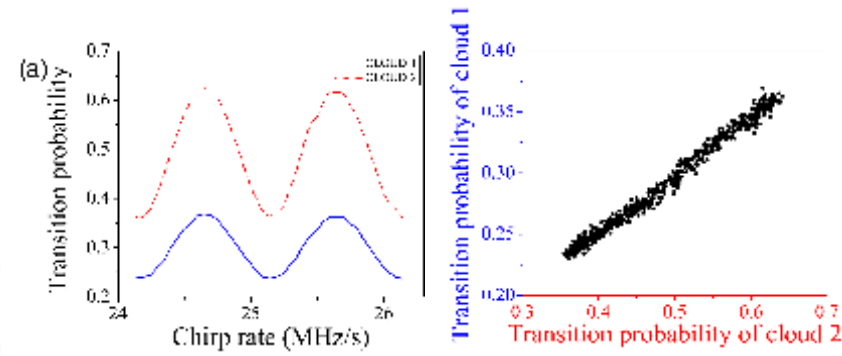
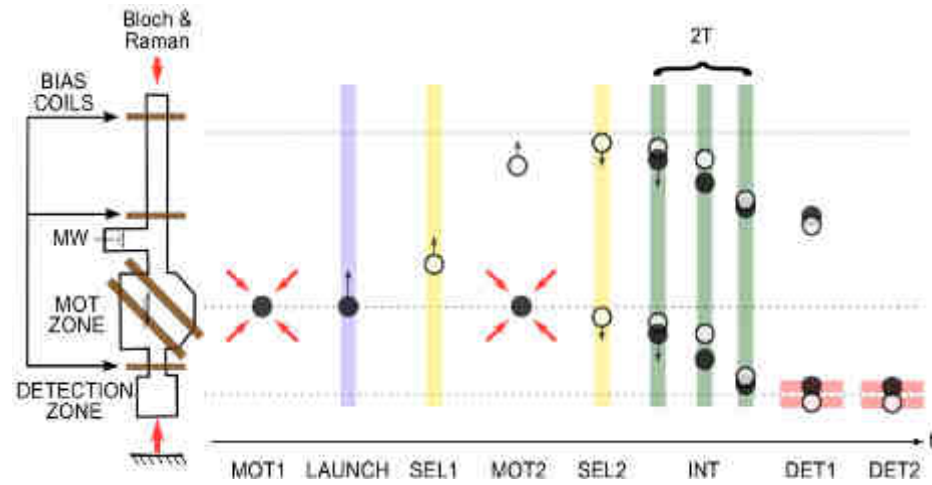
$$\Delta z = 1m$$

$$\sigma_{\text{grad}g} = 126 mE @1s$$

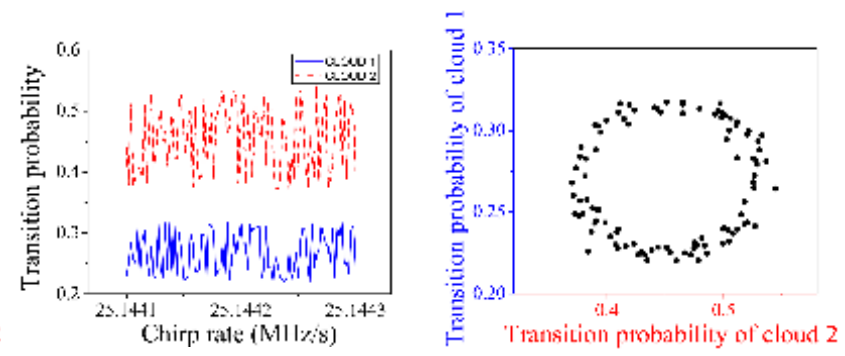
More than one order of magnitude better than state of the art



Quantum dual gravi-radio meter, first results

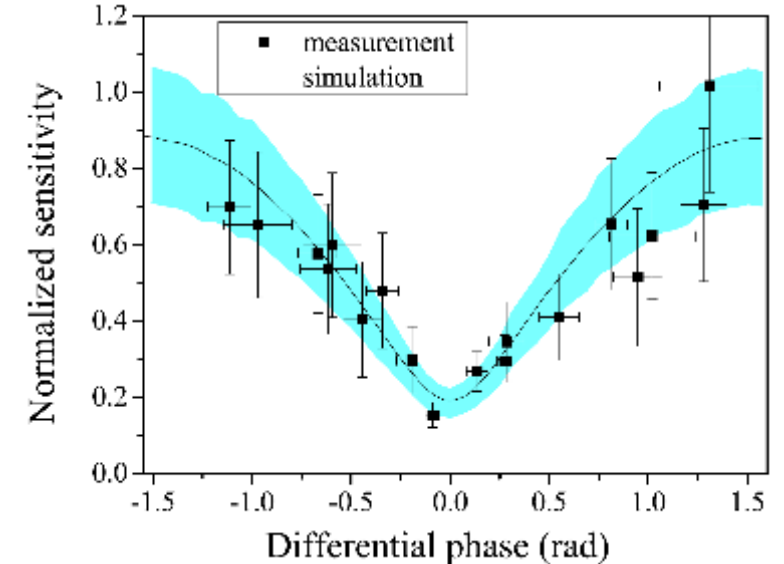
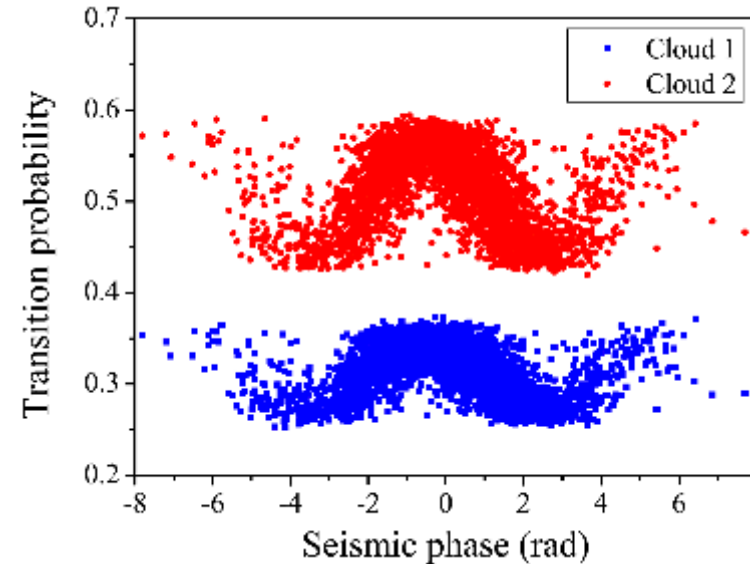


T=1ms



T=35ms

T=60ms

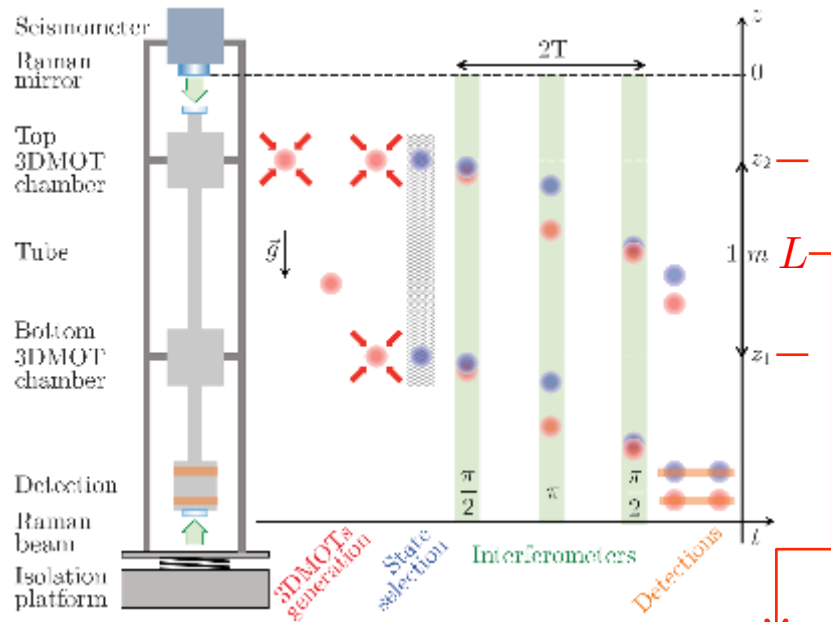


Use of a seismometer

$$P_i = A_i + \frac{C_i}{2} \cos(D_i \Phi_{\text{vib},s} + \Phi_i)$$

M. Langlois et al., Phys. Rev. A 96 053624 (2017)

Quantum dual gravi-radio meter, first results

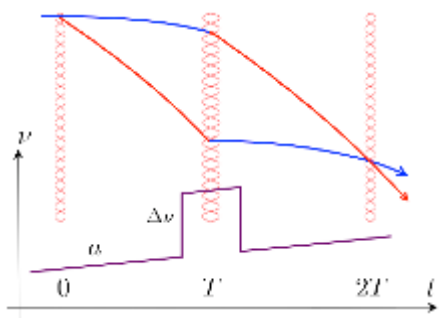


$$\left\{ \begin{array}{l} \Delta\nu = \frac{k(g_1 - g_2)T^2}{K_2 - K_1} \\ a_s = -k \left(\frac{K_2 g_1 - K_1 g_2}{K_2 - K_1} \right) = -k g_s \end{array} \right. \quad \frac{K_i = 8\pi z_i / c}{g_i = g_0 + \gamma z_i} \quad \left\{ \begin{array}{l} \Delta\nu_\gamma = -\gamma \frac{kT^2 c}{8\pi} \\ g_s = g_0 \end{array} \right.$$

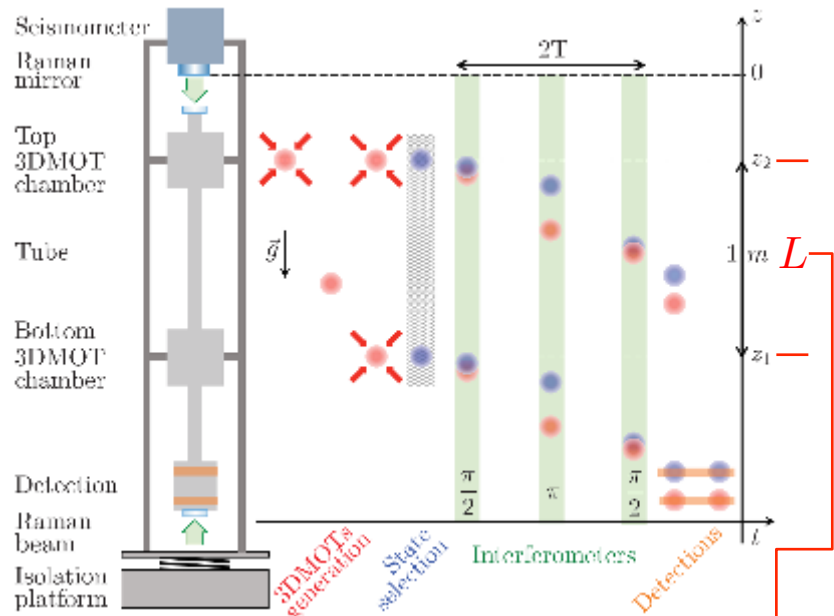
Accurate determinations of **both** the gravity acceleration (at the mirror position) and the gravity gradient, **independent from the baseline**

$$\Delta\Phi = \Phi_2 - \Phi_1 = k g_2 T^2 - k g_1 T^2 = k \gamma \boxed{L} T^2$$

$$\Phi_i = k g_i T^2 + a T^2 + K_i \Delta\nu \quad \left\{ \begin{array}{l} \Delta\Phi_i^{\text{FC}} = a T^2 \\ \Delta\Phi_i^{\text{FJ}} = K_i \Delta\nu \end{array} \right.$$



Quantum dual gravi-radio meter, first results

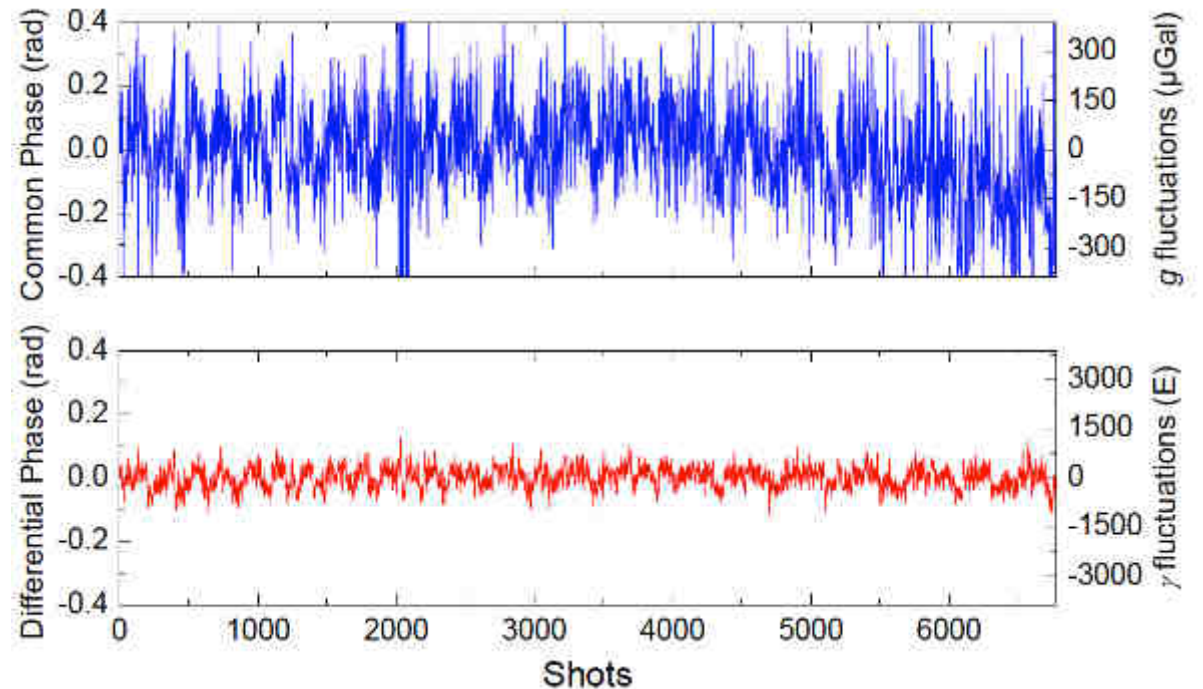
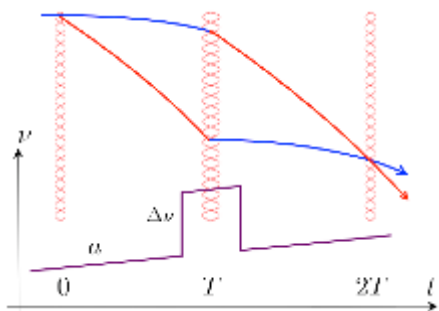


$$\left\{ \begin{array}{l} \Delta\nu = \frac{k(g_1 - g_2)T^2}{K_2 - K_1} \\ a_s = -k \left(\frac{K_2 g_1 - K_1 g_2}{K_2 - K_1} \right) = -k g_s \end{array} \right. \quad \frac{K_i = 8\pi z_i / c}{g_i = g_0 + \gamma z_i} \quad \left\{ \begin{array}{l} \Delta\nu_\gamma = -\gamma \frac{kT^2 c}{8\pi} \\ g_s = g_0 \end{array} \right.$$

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R. Caldani et al., Phys. Rev. A 99 033601 (2019)

Conclusion

Le CAG a démontré sa capacité à mesurer, de manière **absolue, g en continu.**

*g dans $h_{\text{LNE-2017}}$, prise en compte par le CODATA dans l'ajustement h_{2018} pour la nouvelle définition du kg
→ Nouvelle mise en pratique*

Participe à des comparaisons internationales et nationales.

Les gravimètres atomique sont adaptés pour des applications jusqu'alors réalisées par des gravimètres supraconducteurs et/ou à coins de cube en chute libre.

Technologie transférée à l'industrie. Support aux utilisateurs.

Performances encore améliorables avec les atomes ultra-froids,
ouvrir la « sea of problems » et de nouvelles applications. → **objectif sub- μGal**

9.808 907 45(2) m/s^2

Double capteur **Gravi-Gradio** en cours de développement. *La mesure des deux grandeurs, qui dépendent différemment des masses et de leurs positions, permettra de résoudre les ambiguïtés entre les masses et les positions des sources d'anomalies de gravité.*

→ Soutenance de thèse Romain Caldani vendredi 10h

La cartographie gravimétrique embarquée sera bientôt possible avec des gradiomètres à atomes UF pilotés avec des LMBS.

- **Aller dans l'espace ?**
- Atomes sont en chute libre comme le satellite, $T=10s$?; ($2T=5s$ $2mE@1s$)

<https://synte.obsspm.fr/spip/science/iaci/>
<https://synte.obsspm.fr/spip/science/iaci/publications/>

EMN European Metrology Network

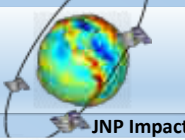
PNTG : Positioning Navigation Timing and Geodesy

Sea level monitoring, future georeferenced services like autonomous driving, classical construction surveying, or time-synchronization in telecommunication networks – they are all examples for economic and scientific applications of internationally highly-coordinated positioning, navigation, timing, and geodesy (PNTG) infrastructure. Advance in these classically separated fields leads to similar metrological challenges and requires interdisciplinary approaches combining gravimetry, time and frequency, and length metrology. A European Metrology Network (EMN) in PNTG seems an ideal vehicle to organize such collaboration efficiently in Europe. This project will lay the groundworks for this EMN.

In summary, the project will reorganize the metrology competence in PNTG, spread today over different locations and metrology disciplines to a joint, focussed, and effective network. The target of the project and all its outcomes is to address the explicit needs in industry and scientific geodesy from the end-user point of view. This will support most critical services for modern economy, as well as for hazard monitoring.

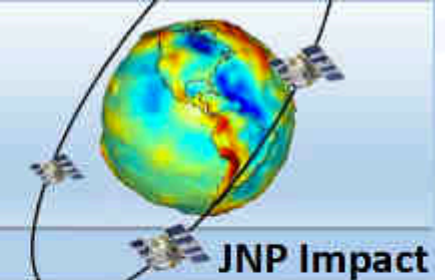
JNP w-02 PNTG

Support for an EMN on Positioning, Navigation, Timing and Geodesy

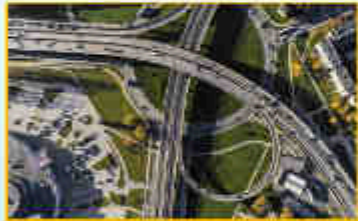


JNP w-02 PNTG

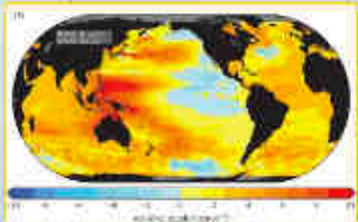
Support for an EMN on Positioning, Navigation, Timing and Geodesy



Need



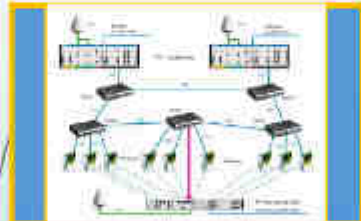
Georeferenced services
Autonomous driving, ...



Sea level monitoring
 Δ height \sim 3.5 mm/yr



Kibble balance
 $U(g) \sim O(10^{-8})$

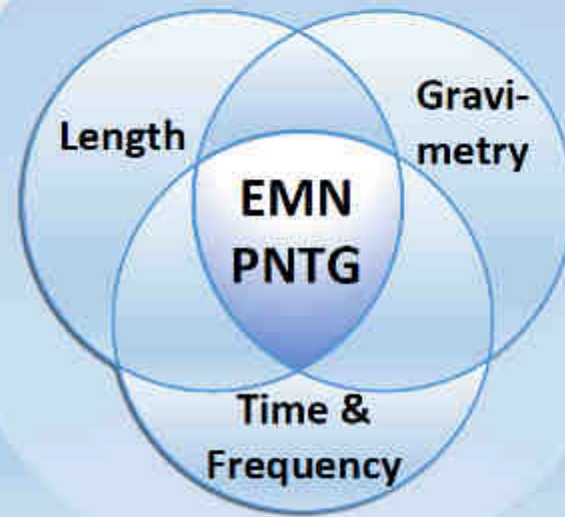


Network synchronization
Finance, telecom, energy



Critical site monitoring
 Δ position \sim 0.1 mm/yr

UN General Assembly 2015: 
„...invites member states to enhance the global geodetic reference frame“



Successful EMN PNTG launch

Strategic Research Agenda (SRA): coordinated, stakeholder-oriented metrology research in the field
Infrastructure plan for efficient PNTG metrology
New structures and tools, e.g.:

- Capacity database:** European metrology experts for IAG, ISO, ITU-T, European Commission
- Service database:** PNTG services European-wide more easily accessible

Wider Impact through the EMN

Advancing metrology for key security infrastructure
Environmental and civil hazard monitoring
Reliability and robustness for „real-time economy“

„Such an **international partner to tackle metrology problems** would reflect the successful international organization and working structure of surveying in FIG ...“
R. Staiger, FIG President

„Although this is a European endeavor, we have joined the program, because **we recognize the global impact ...**“ M. Pearlman, ILRS Central Bureau

„The network would be particularly interesting for the **European GNSS Agency (GSA) and the European GNSS programmes...**“ F. Diani, GSA

„... **information on available facilities, methods and services** for development and testing of

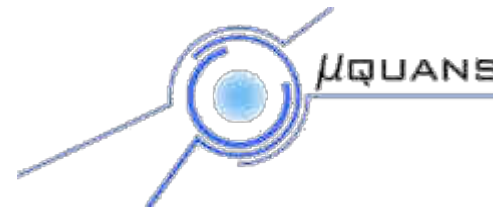
Metrology Competence



JNP Objectives

- Efficient EMN preparation**
- Liaison development —
 - Web PNTG platform —
 - SRA for PNTG metrology —
 - Knowledge-sharing programme —
 - European PNTG metrology research infrastructure plan —

Merci



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