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Overbounding the GNSS Positioning Integrity Risk

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Outline

Two GNSS integrity preservation methods

The GNSS integrity monitoring methods can be broadly divided into two classes: "active integrity methods" and "passive integrity methods".

- For example, the Receiver Autonomous Integrity Monitoring (RAIM) with Fault Detection/Exclusion (FDE) functions belongs to the class of active integrity methods. If an unbounded additional pseudo-range bias in one (or two) GNSS channel(s) occurs at an unknown time then the only solution to preserve a high constant integrity level of GNSS positioning is to use the active integrity methods, like RAIM.
- Degradations of several pseudo-range measurements (additional biases and/or Cumulative Distribution Function (CDF) shape deformation), even when bounded, can lead to unacceptable positioning errors, especially when considering reduced alert limits – like those provided by GBAS, SBAS, and, in the future, ARAIM. A reasonable solution to such a problem consists in the passive integrity method, based on pseudo-ranges "overbounding".

GNSS positioning solution

The linearized pseudo-range equation with respect to the vector X_u around the working point $X_{u\,0} = (x_0, y_0, z_0)^T$ for both single- and dual-frequency measurements

$$R - D_0 \simeq H(X - X_0) + \xi, \quad (1)$$

where $R = (r_1, \ldots, r_m)^T$ denotes the vector of pseudo-range measurements, $D_0 = (d_{1\,0}, \ldots, d_{m\,0})^T$, $d_{i\,0} = \|X_i - X_{u\,0}\|_2$, $X_0 = (X_{u\,0}^T, 0)^T$ and $H = \frac{\partial R}{\partial X}\big|_{X=X_0}$ is a Jacobian matrix of size $(m \times 4)$, and $\xi = (\xi_1, \ldots, \xi_m)^T$ denotes the additive pseudo-range errors at the user's position.

The LS method :

$$\widehat{X} = X_0 + A(R - D_0), \; A = (H^T \Sigma^{-1} H)^{-1} H^T \Sigma^{-1} \quad (2)$$

is the best linear unbiased estimator of X under assumption that $m \geq 5$, $B = \mathbb{E}(\xi) = 0$ and $\mathrm{cov}(\xi) = \Sigma$ is known.

The instantaneous integrity risk

As it follows from (2), the vector of positioning errors $\widehat{X} - X$ (in ENU coordinates) is a linear combination of the pseudo-range errors $\xi_1, \ldots, \xi_m$

$$Q = \widehat{X} - X = A\xi. \tag{3}$$

The instantaneous (per GNSS epoch) integrity risks for the horizontal and vertical positioning are defined by the following probabilities

$$\mathbb{P}\left(\|Q_h\|_2 \geq \mathrm{HAL}\right), \tag{4}$$

where $Q_h = (\widehat{x} - x, \widehat{y} - y)^T = A_h\xi$, A_h is a sub-matrix composed of the first two rows of the matrix A defined in (2) and HAL means the Horizontal Alert Limit, and

$$\mathbb{P}\left(|Q_v| \geq \mathrm{VAL}\right), \tag{5}$$

where $Q_v = \widehat{z} - z = A_v\xi$, A_v is a sub-matrix composed of the third row of the matrix A defined in (2) and VAL means the Vertical Alert Limit.

The integrity risk over a given period of time [Nikiforov, 2019]

The MOPS for GPS/Galileo require calculating the integrity risk over a given period of time (e.g., "per approach" or "per hour"). Let

$$Q_{h,n} = (1-\lambda)Q_{h,n-1} + \lambda A_h \xi_n, Q_{v,n} = (1-\lambda)Q_{v,n-1} + \lambda A_v \xi_n, \quad (6)$$

where $Q_{h,n} = (\widehat{x}_n - x_n, \widehat{y}_n - y_n)^T$, $Q_{v,n} = \widehat{z}_n - z_n$, be the autoregressive model (AR(1)). Let us define the following stopping times N :

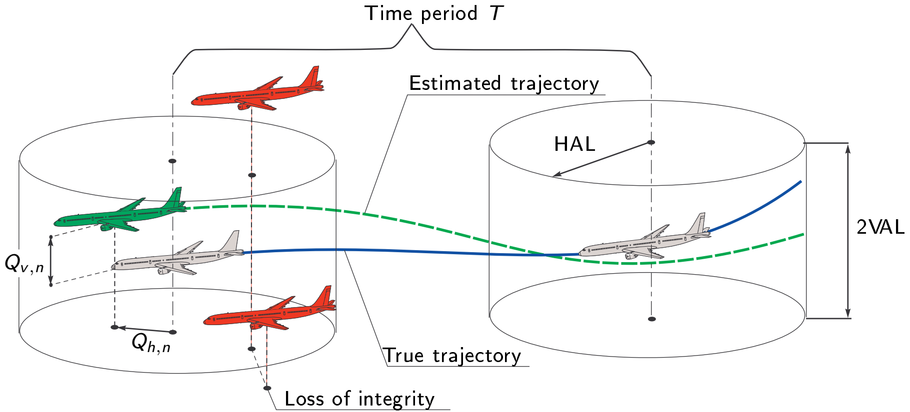
$$N_h = \inf\left\{n \geq 1 : \|Q_{h,n}\|_2 \geq \mathrm{HAL}\right\}, N_v = \inf\left\{n \geq 1 : |Q_{v,n}| \geq \mathrm{VAL}\right\}. \quad (7)$$

The horizontal and vertical integrity risk over a reference period of time T are defined as the conditional probabilities of the events $\{N_h \leq T\}$ and $\{N_v \leq T\}$

$$\mathbb{P}(N_h \leq T \,|\, \|Q_{h,0}\|_2 < \mathrm{HAL}), \mathbb{P}(N_v \leq T \,|\, |Q_{v,0}| < \mathrm{VAL}), \quad (8)$$

provided that $\|Q_{h,0}\|_2 < \mathrm{HAL}$ and $|Q_{v,0}| < \mathrm{VAL}$.

GNSS integrity risk for aircraft navigation



Problem statement

We are interested what happens if

- Pseudo-range error bias is $B = \mathbb{E}(\xi) \neq 0$
- variance-covariance matrix Σ is only partially known
- (and moreover !) the CDFs $F_{\xi,i}(x)$ of ξ_i, $i = 1, \ldots, m$, are unknown and only their upper $\overline{F}_{\xi,i}(x)$ and lower $\underline{F}_{\xi,i}(x)$ bounds (overbounds) are available.

Let us assume that

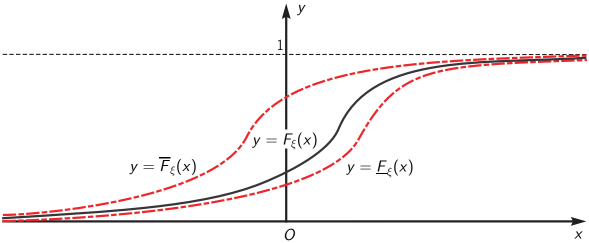
- The estimation $\widehat{X}_n$ of X_n is calculated at each step n
- The autocorrelated positioning errors $Q_1, Q_2, \ldots$ are defined by the AR(1) model

$$Q_n = (1-\lambda)Q_{n-1} + \lambda A\xi_n, \quad n = 1, 2, 3, \ldots. \tag{9}$$

Goal : find the conservative bounds for the instantaneous integrity risk and the integrity risk over a given period of time.

Paired CDF overbounding [Rife, Pullen, Enge and Pervan, 2006]

See [Rife, Pullen, Enge and Pervan, 2006] for details/motivation of the paired CDF overbounding.



Assumption 1 Let us assume that the CDF $F_\xi(X) = \prod_{i=1}^{m} F_{\xi,i}(x_i)$ of the pseudo-range errors $\xi = (\xi_1, \ldots, \xi_m)^T$ obey the following inequality for $i = 1, \ldots, m$

$$\underline{F}_{\xi,i}(x) \leq F_{\xi,i}(x) \leq \overline{F}_{\xi,i}(x) \text{ for } x \in \mathbb{R}.$$

Overbounding the vertical instantaneous integrity risk

The paired CDF overbounding [Rife, Pullen, Enge and Pervan, 2006] is well-adapted to the linear combination of several independent pseudo-range errors $\xi_1, \ldots, \xi_m$:

$$Q_v = \widehat{z} - z = \sum_{i=1}^{m} a_{3,i}\xi_i = A_v \xi. \tag{10}$$

Hence, the conservative vertical instantaneous integrity risk is

$$\mathbb{P}\left(|Q_v| \geq \text{VAL}\right) \leq \overline{p}_1 = 1 - \underline{F}_{Q_v}(\text{VAL}) + \overline{F}_{Q_v}(-\text{VAL}), \tag{11}$$

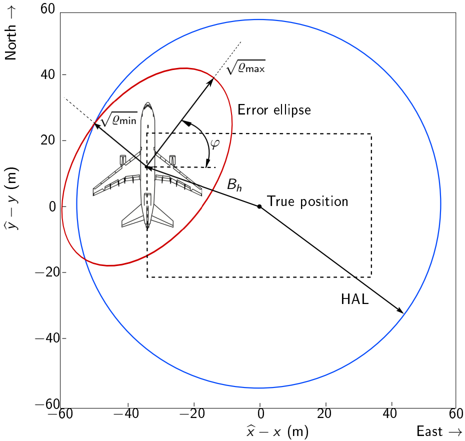
where the bounds $\underline{F}_{Q_v}(x)$ and $\overline{F}_{Q_v}(x)$ are calculated by recursive convolutions of $\underline{F}_{\xi,i}(x)$ and $\overline{F}_{\xi,i}(x)$: $\overline{F}_{Q_v}(x) = (((\overline{f}_1 * \overline{f}_2) * \overline{f}_3) * \cdots * \overline{f}_m)$.
On the contrary, in the horizontal risk overbounding, the radial error $\|Q_h\|_2$ is a nonlinear function of several independent pseudo-range errors $\xi_1, \ldots, \xi_m$:

$$\|Q_h\|_2 = \|(\widehat{x} - x, \widehat{y} - y)\|_2 = \|A_h \xi\|_2. \tag{12}$$

Hence, the paired CDF overbounding cannot be applied directly to the horizontal risk and the integrity risk over a given time period.

Overbounding the horizontal instantaneous integrity risk [Nikiforov, 2019]

Step 1 : The calculation of the conservative bound for the risk due to the bias $B = (b_1, \dots, b_m)^T$ uncertainty.



Let us consider that the pseudo-range errors ξ_i are distributed following the Gaussian distribution $\xi_i \sim \mathcal{N}(b_i, \sigma_i^2)$ and that the absolute value of the bias b_i is upper bounded by $\overline{b}_i$:

$$-\overline{b}_i \leq b_i \leq \overline{b}_i, \quad i = 1, \dots, m. \tag{13}$$

The functions of the pseudo-range overbounding are given by

$$\begin{aligned} \underline{F}_{\xi,i}(x) &= \mathcal{N}(\overline{b}_i, \sigma_i^2) \\ \overline{F}_{\xi,i}(x) &= \mathcal{N}(-\overline{b}_i, \sigma_i^2). \end{aligned} \tag{14}$$

The probability of the event $\{\|Q_h\|_2 \geq \text{HAL}\}$ is given by the function $F_\ell(\text{HAL}^2, \Lambda, \omega)$

$$\begin{aligned} \mathbb{P}\left(\|Q_h\|_2 \geq \text{HAL}\right) &= 1 - F_\ell\left(\text{HAL}^2, \Lambda, \omega\right), \\ F_\ell(y, \Lambda, \omega) &= (2\pi)^{-\frac{\ell}{2}} \int \cdots \int_{\{(W-\omega)^T \Lambda (W-\omega) \leq y\}} \exp\left\{-\frac{1}{2}\|W\|_2^2\right\} dW, \end{aligned} \tag{15}$$

where $\ell = 2$, $W \in \mathbb{R}^\ell$ denotes the support of the Gaussian distribution $\mathcal{N}(0, I_\ell)$ and $\omega = -\Lambda^{-\frac{1}{2}} U^T B_h$.
The analysis of the function $F_2(y, \Lambda, \omega)$ shows that there are two factors determining the probability (15) :

- the vector of systematic horizontal errors $B_h \in \mathbb{R}^2$;
- the orientation φ of the error ellipse with respect to the West - East axis. The angle φ is a function of the variance-covariance matrix Σ.

The vector of systematic horizontal errors B_h can be expressed as a linear function of the vector of pseudo-range biases B, i.e., $B_h = A_h B$. Let us define the following hyperrectangle $\mathbb{B} = \left\{X \in \mathbb{R}^m | x_i \in [-\overline{b}_i, \overline{b}_i],\ i = 1, \ldots, m\right\}$ and a linear mapping (defined by the matrix A_h) of the set $\mathbb{B}$ onto the set $\mathbb{P}$. The set $\mathbb{P}$ is a convex polygon.

$$\max_{B \in \mathbb{B}} \mathbb{P}(\|Q_h\|_2 \geq \mathrm{HAL}) = 1 - \min_{B \in \mathbb{B}} F_2\left(\mathrm{HAL}^2, \Lambda, -\Lambda^{-\frac{1}{2}} U^T A_h B\right). \tag{16}$$

To reduce computational burden, the error ellipse can be overestimated by a disk of the radius $\sqrt{\varrho_{\max}}$, where $\varrho_{\max} = \max\{\varrho_1, \varrho_2\}$ and ϱ_1, ϱ_2 are eigenvalues of the matrix Σ.

$$\begin{aligned} \max_{B \in \mathbb{B}} \mathbb{P}\left(\|Q_h\|_2 \geq \mathrm{HAL}\right) &\leq 1 - \min_{B \in \mathbb{B}} F_2\left(\mathrm{HAL}^2, \overline{\Lambda}, -\overline{\Lambda}^{-\frac{1}{2}} A_h B\right) \\ &\leq 1 - F_2\left(\mathrm{HAL}^2, \overline{\Lambda}, -\overline{\Lambda}^{-\frac{1}{2}} \overline{B}_h\right) \\ &\leq 1 - F_2\left(\mathrm{HAL}^2, \overline{\Lambda}, -\overline{\Lambda}^{-\frac{1}{2}} B_h^*\right), \end{aligned} \tag{17}$$

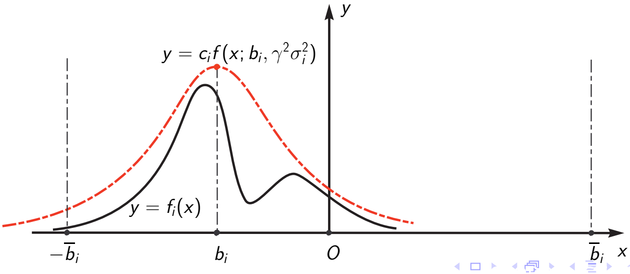
where $\overline{\Lambda} = \mathrm{diag}\{\varrho_{\max}, \varrho_{\max}\}$, $\overline{B}_h = A_h B_j$, $j = \arg\max_{i=1,\ldots,2^m} \{\|A_h B_i\|_2\}$, and B_i is a vertex of $\mathbb{B}$, $i = 1, \ldots, 2^m$, $B_h^* = \left(\sum_{i=1}^m |a_{1,i}|\overline{b}_i \quad \sum_{i=1}^m |a_{2,i}|\overline{b}_i\right)^T$.

Step 2 : Let $B = (b_1, \ldots, b_m)^T$ be such that $B \in \mathbb{B}$. The PDF $f_{\xi,i}(x)$ of the pseudo-range errors ξ_i, $i = 1, \ldots, m$, is upper bounded by the PDF $f(x; b_i, \gamma^2\sigma_i^2)$ of the Gaussian law $\mathcal{N}(b_i, \gamma^2\sigma_i^2)$ with the coefficient of inflation c_i and the sigma-inflation $\gamma \geq 1$ ("Excess-Mass PDF overbounding" proposed in [Rife, Walter and Blanch, 2004]) :

$$f_{\xi,i}(x) \leq c_i f(x; b_i, \gamma^2\sigma_i^2) \text{ for } x, b_i \in \mathbb{R}, \quad i = 1, \ldots, m. \tag{18}$$

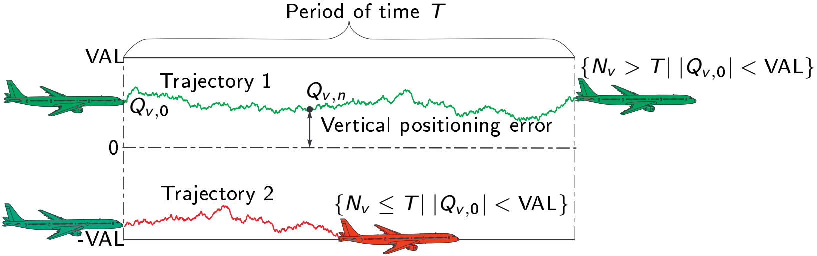
Finally, the simplified overbounding formula is

$$\mathbb{P}(\|Q_h\|_2 \geq \text{HAL}) \leq \left[\prod_{i=1}^{m} c_i\right]\left[1 - F_2\left(\text{HAL}^2, \gamma^2\overline{\Lambda}, -\gamma^{-1}\overline{\Lambda}^{-\frac{1}{2}} B_h^*\right)\right]. \tag{19}$$



Overbounding the vertical integrity risk over a given period of time [Nikiforov, 2019]

Let us calculate a conservative bound for the conditional probability of the event $\{N_v \leq T| \, |Q_{v,0}| < \text{VAL}\}$ provided that $|Q_{v,0}| < \text{VAL}$:



- by solving integral equations [Kemperman, 1950, Page, 1954];
- first-passage-problem [Cox and Miller, 1965, Ch. 2];
- AR(1) [Crowder, 1987] and [Nikiforov, 2017a];
- by level-crossing problem [Rice 1944, Rice 1945] and [Cramér and Leadbetter, 1967, Leadbetter, Lindgren, and Rootzén, 1983].

Assumption 2 : Let us assume that the CDF $F_{Q_0}(X) = \prod_{i=1}^{m} F_{Q_0,i}(x_i)$ of the initial state Q_0 obey the following inequality for $i = 1, \ldots, m$

$$\underline{F}_{Q_0,i}(x) \leq F_{Q_0,i}(x) \leq \overline{F}_{Q_0,i}(x) \text{ for } x \in \mathbb{R}.$$

Step 1 : Let us consider that Assumptions 1 and 2 are satisfied. Then the upper bound $\overline{p}_n(u)$ for the probability $p_n(u) = \mathbb{P}(N_v = n | Q_{v,0} = u)$ is given by

$$\begin{aligned} p_n(u) \leq \overline{p}_n(u) &= \overline{p}_{n-1}(h)\overline{F}_y\left(\frac{h-(1-\lambda)u}{\lambda}\right) - \overline{p}_{n-1}(-h)\underline{F}_y\left(\frac{-h-(1-\lambda)u}{\lambda}\right) \\ &- \int_{-h}^{h} \underline{F}_y\left(\frac{z-(1-\lambda)u}{\lambda}\right)\mathbb{1}_{\{\overline{p}'_{n-1}(z)\geq 0\}}\overline{p}'_{n-1}(z)dz \\ &- \int_{-h}^{h} \overline{F}_y\left(\frac{z-(1-\lambda)u}{\lambda}\right)\mathbb{1}_{\{\overline{p}'_{n-1}(z)< 0\}}\overline{p}'_{n-1}(z)dz, \end{aligned} \tag{20}$$

where $n = 2, 3, \ldots, T$, $\mathbb{1}_{\{A\}}$ is the indicator function of A, $\overline{p}'_{n-1}(z) = d\overline{p}_{n-1}(z)/dz$ and the upper bound for the probability $p_1(u)$ is given by

$$\overline{p}_1(u) = 1 - \underline{F}_y\left(\frac{h-(1-\lambda)u}{\lambda}\right) + \overline{F}_y\left(\frac{-h-(1-\lambda)u}{\lambda}\right). \tag{21}$$

Step 2 : Let us consider that Assumptions 1 and 2 are satisfied. The initial condition $Q_{v,0} = u$ follows $F_{Q_{v,0}}$, we have to randomize the result in the following manner (under assumption that $u \in]-h, h[$) :

$$p_r = \mathbb{P}(N_v \leq T | u \in]-h, h[) = \frac{\int_{-h}^{h} f_{Q_{v,0}}(x) p_T(x) dx}{\int_{-h}^{h} f_{Q_{v,0}}(x) dx}, \quad (22)$$

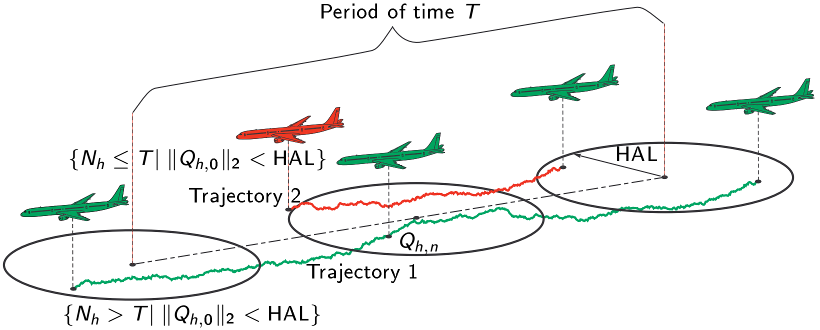
where $p_T(u) = \mathbb{P}(N_v \leq T | Q_{v,0} = u) = \sum_{n=1}^{T} p_n(u)$, $u \sim F_{Q_{v,0}}$, $f_{Q_{v,0}}(x)$ is the PDF of $F_{Q_{v,0}}$. Then the upper bound $\overline{p}_r$ for the vertical integrity risk per a given period of time $p_r = \mathbb{P}(N_v \leq T | u \in]-h, h[)$ is given by

$$p_r \leq \overline{p}_r = \frac{1}{a} \Big[\overline{p}_T(h) \overline{F}_{Q_{v,0}}(h) - \overline{p}_T(-h) \underline{F}_{Q_{v,0}}(-h) \quad (23)$$
$$- \int_{-h}^{h} \underline{F}_{Q_{v,0}}(x) \mathbb{1}_{\{\overline{p}'_T(x) \geq 0\}} \overline{p}'_T(x) dx - \int_{-h}^{h} \overline{F}_{Q_{v,0}}(x) \mathbb{1}_{\{\overline{p}'_T(x) < 0\}} \overline{p}'_T(x) dx \Big],$$

where $h = \mathrm{VAL}$, $a = \underline{F}_{Q_{v,0}}(h) - \overline{F}_{Q_{v,0}}(-h)$, $\overline{p}_T(x) = \sum_{n=1}^{T} \overline{p}_n(x)$ and $\overline{p}'_T(x) = d\overline{p}_T(x)/dx$.

Overbounding the horizontal integrity risk over a given period of time [Nikiforov, 2019]

Let us calculate a conservative bound for the conditional probability of the event $\{N_h \leq T | \, \|Q_{h,0}\|_2 < \text{HAL}\}$ provided that $\|Q_{h,0}\|_2 < \text{HAL}$:

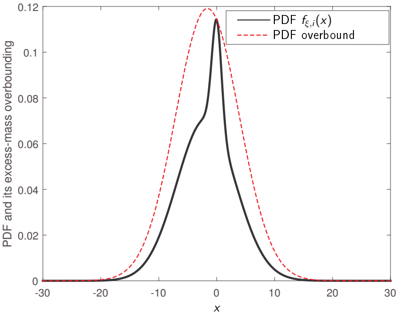


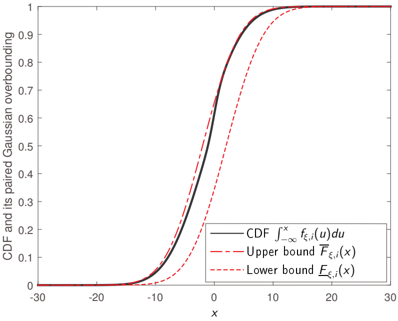
Solution : the same method as previously, by the passage from a simple integral to a double integral. See details in [Nikiforov, 2019].

Examples [Nikiforov, 2019]

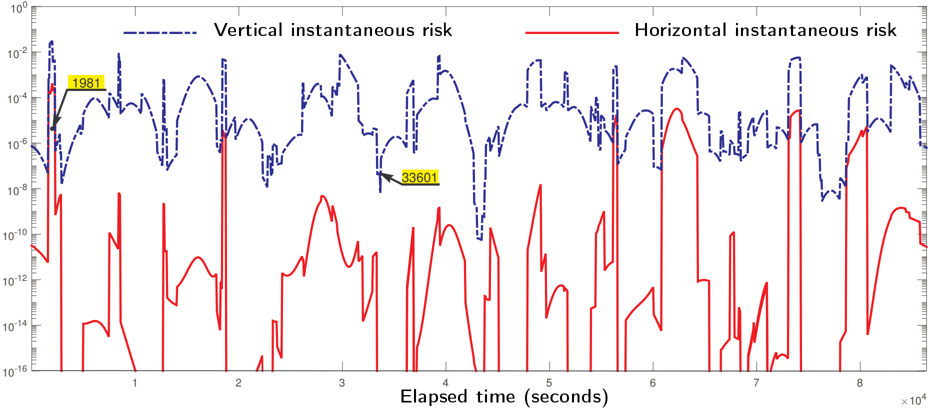
- The GPS constellation is simulated using GPS week 0593 (Jan. 2011).
- The LPV-200 mode of flight, HAL $= 40$ m and VAL $= 35$ m.
- The probability of HMI (Hazardously Misleading Information) is upper bounded by $2 \cdot 10^{-7}$ per approach ($T = 150$ seconds)
- The covariance matrix is $\Sigma = \mathrm{diag}\left\{\sigma_1^2, \ldots, \sigma_m^2\right\} = \mathrm{diag}\left\{25, \ldots, 25\right\} \mathrm{m}^2$.
- The pseudo-range biases b_i are bounded by $\overline{b} = 2$ m, $i = 1, \ldots, m$.
- The user's coordinates $(\phi, \lambda, h) = (48^\circ\ 16'\ 7'', 4^\circ\ 3'\ 57'', 178\ \mathrm{m})$.
- The elevation mask angle is set to 7°.
- Two methods of the pseudo-range error overbounding are used :
  - $f_{\xi,i}(x) \leq cf(x; b_i, \gamma^2\sigma_i^2)$ for $x \in \mathbb{R}$, $\quad i = 1, \ldots, m$,
  - $\underline{F}_{\xi,i}(x) = \mathcal{N}\left(\overline{b}, \sigma_i^2\right) \leq F_{\xi,i}(x) \leq \overline{F}_{\xi,i}(x) = \mathcal{N}\left(-\overline{b}, \sigma_i^2\right)$.
- $f_{\xi,i}(x) = \omega f(x; -\overline{b}, \sigma_i^2) + 0.1\ f(x; 0, \sigma_i^2/36) + (1 - 0.1 - \omega)\ f(x; \overline{b}, \sigma_i^2)$, $i = 1, \ldots, m$, where $\omega \in [0.07, 0.83]$.

The tuning parameters $c = 1.64$ and $\gamma = 1.1$ of the excess-mass PDF overbounding are chosen as the minimum inflation coefficients such that inequality is satisfied for all possible $\omega \in [0.07, 0.83]$. This situation is illustrated in the figure for $\omega = 0.83$, which corresponds to the worst case bias $\mathbb{E}(\xi_i)$ of the pseudo-range errors. The choice $\omega = 0.83$ is motivated by the fact that such a PDF/CDF corresponds to the limit positions simultaneously achievable by the two above-mentioned types of the pseudo-range error overbounding.



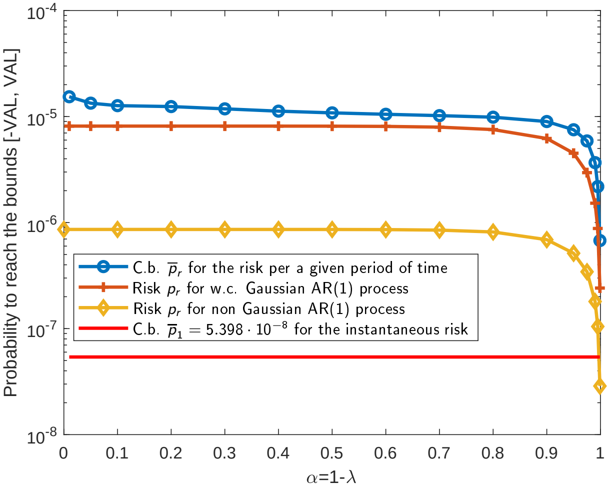


Conservative bounds for the instantaneous risks

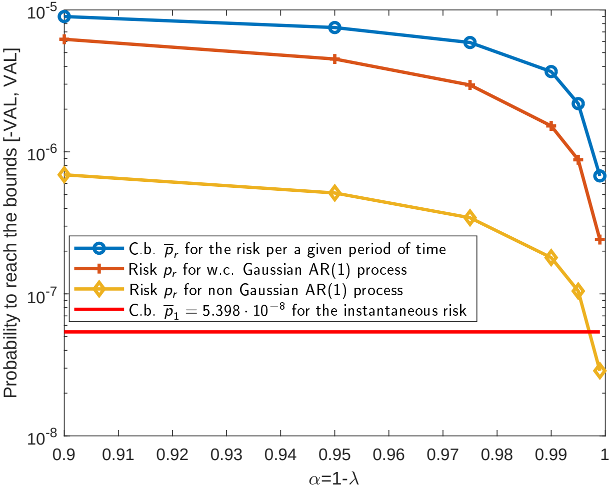


- Vertical instantaneous risk : equation (11)
- Horizontal instantaneous risk : equation (19)

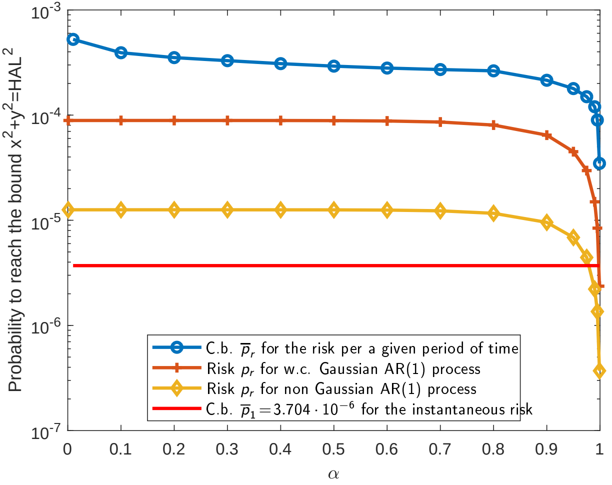
Overbounding the vertical integrity risk over a given period of time



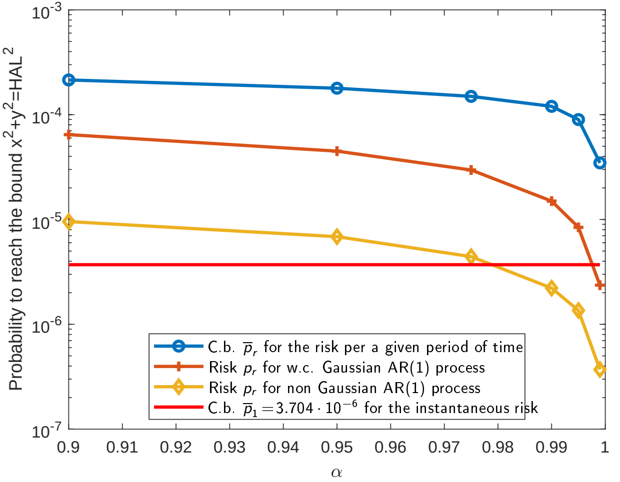
- GPS week 0593 (Jan. 2011).
- Time 33601 seconds.
- The LPV-200 flight operation, VAL $= 35$ m.
- Risk per approach $T = 150$ seconds.
- Probability of HMI $\leq 2 \cdot 10^{-7}$ per approach.
- Sampling period 1 second.


Probability to reach the bounds [-VAL, VAL]
10^{-5}
10^{-6}
10^{-7}
10^{-8}
0.9 0.91 0.92 0.93 0.94 0.95 0.96 0.97 0.98 0.99 1
α=1-λ
C.b. $\overline{p}_r$ for the risk per a given period of time
Risk p_r for w.c. Gaussian AR(1) process
Risk p_r for non Gaussian AR(1) process
C.b. $\overline{p}_1 = 5.398 \cdot 10^{-8}$ for the instantaneous risk

Overbounding the horizontal integrity risk over a given period of time



- GPS week 0593 (Jan. 2011).
- Time 1981 seconds.
- The LPV-200 flight operation, HAL $=$ 40 m.
- Risk per approach $T = 150$ seconds.
- Probability of HMI $\leq 2 \cdot 10^{-7}$ per approach.
- Sampling period 1 second.



10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
Probability to reach the bound $x^2+y^2=HAL^2$
0.9 0.91 0.92 0.93 0.94 0.95 0.96 0.97 0.98 0.99 1
α
C.b. $\overline{p}_r$ for the risk per a given period of time
Risk p_r for w.c. Gaussian AR(1) process
Risk p_r for non Gaussian AR(1) process
C.b. $\overline{p}_1 = 3.704 \cdot 10^{-6}$ for the instantaneous risk

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